Black Holes and AdS/CFT

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AdS/CFT has played an important role in developing our understanding of string theory.

A large part of the studies in AdS/CFT correspondence involves black holes in some way.

In particular in this conference there have already been talks on the application of AdS/CFT to the study of hydrodynamics of strongly coupled gauge theories.

These studies involve black holes in an essential way.

In view of this I shall focus this talk on a different aspect of the relation between black holes and AdS/CFT correspondence.

Extremal Black Holes and AdS_2/CFT_1

This is perhaps the least understood case of AdS/CFT correspondence.

We shall study two aspects:

1. Quantum entropy function

A.S., arXiv:0809.3304 + · · ·

2. Kerr/CFT correspondence.

Guica, Hartman, Song, Strominger, arXiv:0809.4266

+ · · ·

Introduction

One of the successes of string theory has been an explanation of the Bekenstein-Hawking entropy of a class of supersymmetric black holes in terms of microscopic quantum states.

$$S_{BH}(Q) = \ln d_{micro}(Q)$$

Strominger, Vafa

$d_{micro}(Q)$: degeneracy of microstates carrying a given set of charges Q

$$S_{BH}(Q) = A/4G_N$$

This formula is quite remarkable since it relates a geometric quantity in space-time to a counting problem.

One would clearly like to have a better understanding of this corresponence / find generalizations.

We shall study this question in the context of extremal, ı.e. zero temperature black holes.

Since they do not Hawking radiate, the notion of degeneracy is better defined for these black holes.

Often, but not always, they preserve part of the supersymmetry, and hence are stable.

What kind of qusetions would we like to address?

1. The Bekenstein-Hawking formula needs generalization in string theory where the action contains higher derivative corrections and quantum corrections.

The effect of higher derivative corrections are taken into account in a more general formula due to Wald, but what about quantum corrections?

– Goal of the quantum entropy function formalism.

This is necessary if we want to make a precision comparison between the microscopic and the macroscopic entropies of the black hole.

An example: Spectrum of a class of supersymmetric states in heterotic string theory compactified on a six dimensional torus.

charge^2	degeneracy $\mathbf{d}_{\mathbf{micro}}$	$\ln \mathbf{d}_{\mathbf{micro}}$	S_{BH}
2	50064	10.82	6.28
4	32861184	17.31	12.57
6	16193130552	23.51	18.85
8	7999169992704	29.71	25.13
10	4074192429737760	35.943	31.42

2. Understand the entropy of extremal non-supersymetric black holes *e.g.* extreme Kerr black hole from a microscopic viewpoint.

– Goal of Kerr/CFT correspondence.

Eventually we would like to use these as the starting point for studying the thermodynamics of non-extremal black holes.

In both studies AdS_2/CFT_1 correspondence will play an important role.

What is AdS_2?

AdS_2 may be regarded as a two dimensional Lorentzian space embedded in a 3-dimensional space of signature (+,-,-) via the relation:

$$x^2 - y^2 - z^2 = -a^2$$

a: some constant giving the radius of AdS_2.

This space has an SO(2,1) isometry.

Why AdS_2?

All known extremal black holes have an AdS_2 factor in their near horizon geometry.

– time translation symmetry gets enhanced to $SO(2,1)$ in the near horizon limit.

Reissner-Nordstrom solution in $D = 4$:

$$
\begin{aligned}
ds^2 &= -(1-\rho_+/\rho)(1-\rho_-/\rho)d\tau^2 \\
&\quad + \frac{d\rho^2}{(1-\rho_+/\rho)(1-\rho_-/\rho)} \\
&\quad + \rho^2(d\theta^2 + \sin^2\theta d\phi^2)
\end{aligned}
$$

Define

$$
2\lambda = \rho_+ - \rho_-, \quad t = \frac{\lambda\tau}{\rho_+^2}, \quad r = \frac{2\rho - \rho_+ - \rho_-}{2\lambda}
$$

and take $\lambda \to 0$ limit keeping r, t fixed.

$$
ds^2 = \rho_+^2\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right] + \rho_+^2(d\theta^2 + \sin^2\theta d\phi^2)
$$

$$
AdS_2 \qquad \times \qquad S^2
$$

Postulate: Any extremal black hole has an AdS_2 factor / $SO(2,1)$ isometry in the near horizon geometry.

– partially proved

Kunduri, Lucietti, Reall; Figueras, Kunduri, Lucietti, Rangamani

This simple postulate leads to non-trivial consequences.

1. It gives a proof of attractor mechanism in any classical theory of gravity with or without higher derivative terms.

The entropy of an extremal black hole depends only on its charges and not on any other asymptotic data, e.g. the vacuum expectation value of various moduli scalar fields.

2. For spherically symmetric extremal black holes this gives a simple method for computing the entropy without solving any differential equation.

Review of **AdS/CFT** Correspondence

We begin by describing the basic observations / postulates leading to the AdS/CFT correspondence.

1. The boundary of euclidean AdS_{d+1} is a d-dimensional sphere S^d.

2. Given a string theory in euclidean AdS_{d+1} there is an associated d dimensional euclidean CFT on S^d such that

$$Z_{gravity} = Z_{CFT}$$

$Z_{gravity}$: result of path integral of the string theory on AdS_{d+1}.

Z_{CFT}: result of the path integral over the CFT fields on S^d.

3. Often the AdS_{d+1} background arises as the near horizon geometry of an extremal black brane.

In this case there is a simple way to identify the dual CFT.

It is the theory obtained by taking the low energy limit of the quantum theory on the brane system that produces the black hole geometry.

We shall now apply this to AdS_2/CFT_1 correspondence.

Quantum entropy function

Observations:

1. In all known examples in string theory the microscopic system that produces the black hole has an energy gap that separates the ground states from the first excited state.

For fixed values of charges, only the ground states survive in the low energy limit .

2. For $d = 1$, S^d is a circle S^1 of some lenth L.

$$\Rightarrow \qquad Z_{CFT} = Tr(e^{-LH}) = d_{micro}$$

d_{micro}: ground state degeneracy

AdS_2/CFT_1 correspondence now implies:

$$Z_{gravity} = Z_{CFT} = d_{micro}$$

We declare $\ln Z_{gravity}$ to be the quantum generalization of the black hole entropy.

– Quantum entropy function.

The equality between black hole entropy and $\ln d_{micro}$ now becomes a consequence of AdS_2/CFT_1 correspondence.

Consistency check: In the classical limit this reduces to the exponential of the Wald entropy.

This exact formula serves as a starting point for computing systematic corrections to the black hole degeneracy and comparing it with the microscopic result where the result is known.

There has been several checks including comparison of some non-perturbative corrections on both sides.

However lot more work is still left to be done to check the complete equality between $Z_{gravity}$ and d_{micro}.

Kerr/CFT correspondence

Like all extremal black holes, extremal rotating black holes also have AdS_2 factor in their near horizon geometry.

Thus we might expect to apply AdS_2/CFT_1 correspondence for studying these black holes.

However here we proceed somewhat differently.

In conventional approach to defining the quantum theory on AdS_{d+1} we choose the asymptotic boundary condition on different fields in such a way that asymptotic structure of space-time remains unmodified.

In AdS_2 this essentially forces the system to be in its ground state, carrying a fixed set of charges.

In the study of Kerr/CFT correspondence we relax this and allow for a more general class of field configurations.

For this more general class of boundary conditions we can find a set of general coordinate transformations which preserve the boundary condition and generate a Virasoro algebra.

The central charge of the algebra is

$$\mathbf{c} = \mathbf{12J}/\hbar$$

J: classical angular momentum.

Extremal black holes have zero temperature.

However Kerr black holes have two isometries, – time translation and rotation – and hence one can associate two ‘temperatures’ to this black hole.

The temperature associated with one linear combination of the two isometries must vanish due to the extremality condition.

However the ‘temperature’ associated with the other linear combination is non-zero and takes the value $1/2\pi$.

Using the information about the central charge and the temperature one can calculate the entropy.

$$\text{Entropy} = \frac{\pi^2}{3}\, c\, T = \frac{\pi^2}{3}\, \frac{12J}{\hbar}\, \frac{1}{2\pi} = \frac{2\pi J}{\hbar}$$

This agrees with the Bekenstein-Hawking entropy.

This provides a possible understanding of the entropy of a Kerr black hole as the thermal entropy of a dual CFT.

– Kerr/CFT correspondence.

This analysis has been generalized to theories other than pure Einstein gravity.

Final comments

AdS_2/CFT_1 is the least understood of all the dualities.

However this is the duality that allows us to focus directly on extremal black holes for which the microscopic side of the story is much better understood.

Thus a deeper understanding of the AdS_2/CFT_1 correspondence is likely to shed further light on the correspondence between the microscopic entropy and the black hole entropy.

Conversely, tests of equality between black hole entropy and $\ln d_{micro}$ can be used to provide precision tests of AdS_2/CFT_1 correspondence.

A (not completely systematic) one loop calculation gives the following result for the entropy of black holes in heterotic string theory on a six-torus:

charge^2	degeneracy $\mathbf{d_{micro}}$	$\ln \mathbf{d_{micro}}$	S_{BH}	S_{1-loop}
2	50064	10.82	6.28	10.62
4	32861184	17.31	12.57	16.90
6	16193130552	23.51	18.85	23.19
8	7999169992704	29.71	25.13	29.47
10	4074192429737760	35.943	31.42	35.75

Cardoso, de Wit, Kapelli, Mohaupt; David, Jatkar, A.S.