Duality cascade of softly broken supersymmetric theories

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1.Introduction

Supersymmetry (SUSY): good candidate for new physics.

- Solution of fine tuning problem
- Gauge coupling unification, dark matter (with R-parity).

For this talk, a good point of SUSY is having

approaches to strong coupling gauge theory:

Seiberg duality

Motivation (why we want to look at duality)

Building models of particle physics based on the string theory.

Heckman, Vafa, Verlinde, Wijnholt

Because Ramond-Ramond tadpole (anomaly cancellation) condition often requires many D-branes of O(10 - 1000): we tend to have very large gauge group naturally.

Question:

We may find SU(100) gauge group. ⟷ Standard model (SM) SU(3)×SU(2)×U(1).

How can we get small rank of gauge group naturally?

(usually we choose small gauge group by hand.)

Seiberg duality cascade

Klebanov and Strassler, Strassler

Consider $\mathcal{N}=1$ SUSY gauge theory with IR fixed points

$$SU(kN)\times SU((k-1)N), \qquad SU((k-2)N)\times SU((k-1)N).$$

	$SU(kN)$	$SU((k-1)N)$
Q_r	kN	$\overline{(k-1)N}$
$\bar{Q}_s$	$\overline{kN}$	$(k-1)N$

$r,s=1,2$

	$SU((k-2)N)$	$SU((k-1)N)$
q_r	$(k-2)N$	$\overline{(k-1)N}$
$\bar{q}_s$	$\overline{(k-2)N}$	$(k-1)N$

$r,s=1,2$

Seiberg dual for SU(kN)

$$W = h \det_{r,s}(Q_r\bar{Q}_s) \longrightarrow$$

In IR regime:

$$g_k \approx g_k^*,$$
$$g_{k-1} \approx 0, h \approx 0.$$

$$W_{low} = -\frac{y_*^2}{m}\det_{r,s}(q_r\bar{q}_s) \equiv \tilde{h}\det_{r,s}(q_r\bar{q}_s).$$

We have similar model but smaller gauge group!

Duality cascade will continue repeatedly until theory is confined in this Klebanov-Strassler model.

$$SU(kN) \times SU((k-1)N) \to SU((k-1)N) \times SU((k-2)N) \to \ldots$$
$$\cdots \to SU(2N) \times SU(N) \to \text{pure}\, SU(N) \to \text{confined}.$$

Perhaps, more complicated duality cascade leads to the standard model. (But no one still find it.)

$$\prod_i SU(N_i) \stackrel{?}{\to} SU(3) \times SU(2) \times U(1) \times \ldots$$

Rank = O(100)

Toy model:

$$U(3) \times USp(6)^2_{L/R} \times U(1) \quad \to \quad U(3) \times USp(2)^2_{L/R} \times U(1).$$

2 times duality

This is a motivation to the duality cascade.

Furthermore, when one adds SUSY breaking terms to this model, it may become more realistic because it seems that SUSY is broken in our world.

Trial for an explicit model: Uranga et al., Heckman et al. (Seiberg dual: Abel et al., Oz et al.)

Because we want to know just qualitative properties of duality cascade, in this talk we will consider

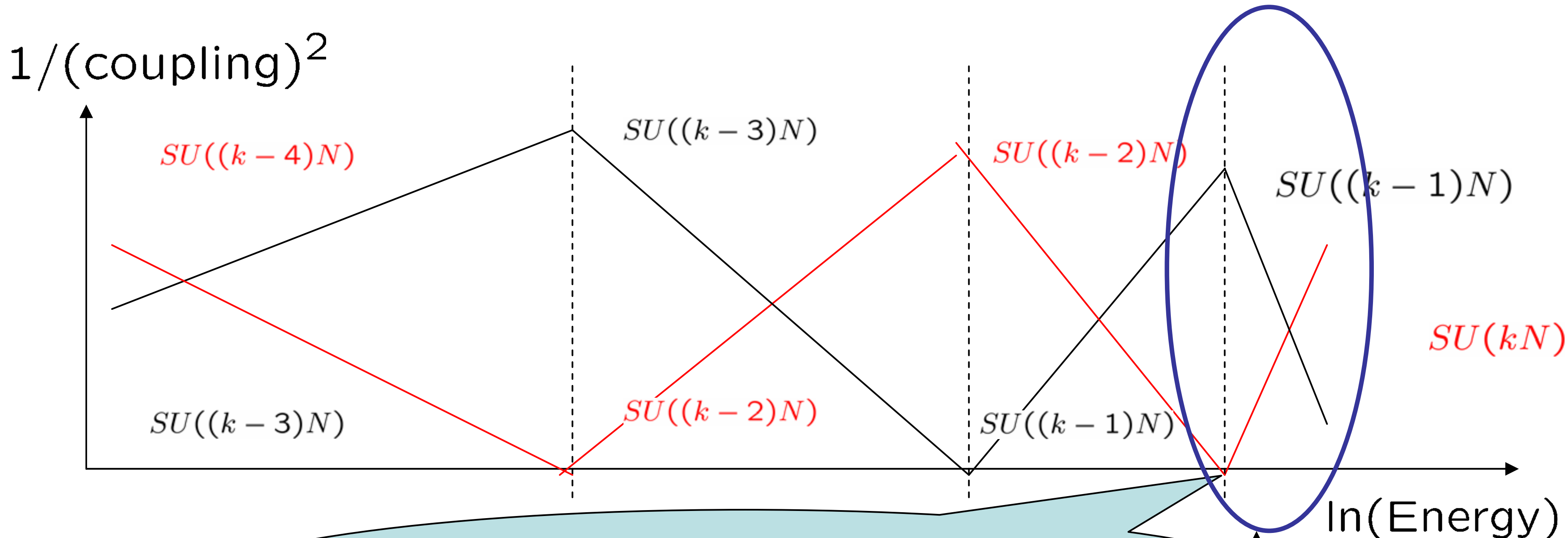
SU(kN)×SU((k-1)N) model with soft SUSY breaking terms

as a first step.

Using a spurion method (physical parameters are treated as external superfields), we will introduce soft SUSY breaking terms by hand.

2. RG flow of gauge couplings
(Reminder)

Rough illustration of running of gauge couplings

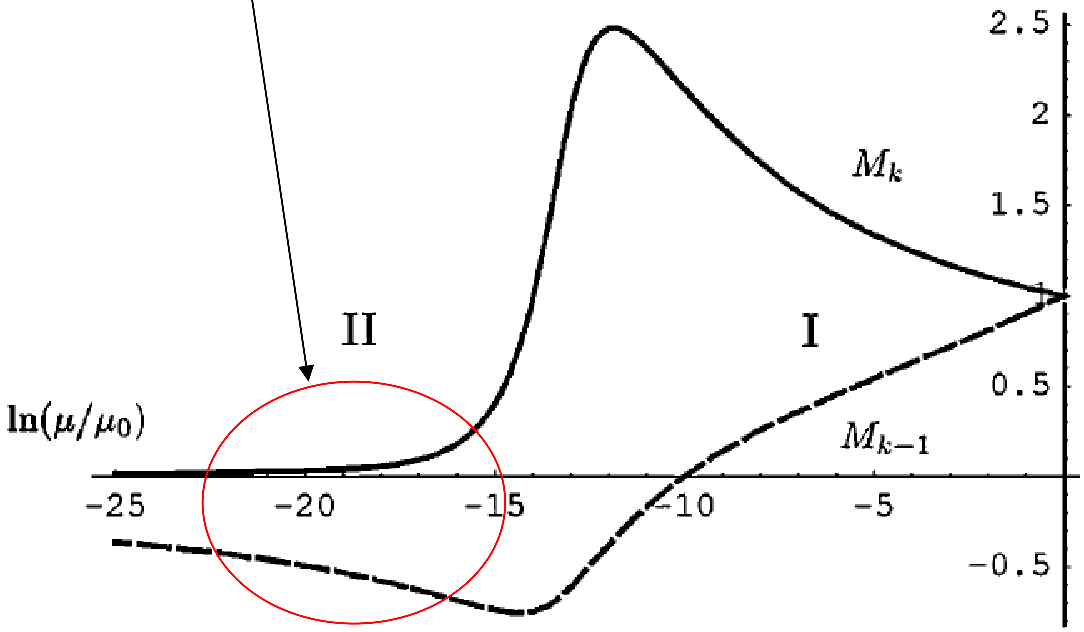


Fixed point: we use Seiberg duality and then integrating out meson; $m/\mu \to \infty$.

For soft mass, we checked mainly and generalized to the another steps.

3. RG equations of soft masses in original theory gaugino masses:

$$\frac{dM_k}{d\ln(\mu)} \sim 0 \quad \xrightarrow{\text{1-loop anomalous dim.}} \quad k\alpha_k^* M_k \sim -(k-1)\alpha_{k-1}M_{k-1}$$



$$\mu\frac{dM^{(k)}_{1/2}}{d\mu} = -N(k+2+2(k-1)\gamma_Q)H'(\alpha_k)\alpha_k M^{(k)}_{1/2} - 2(k-1)NH(\alpha_k)\frac{\partial\gamma_Q}{\partial\alpha_k}\alpha_k M^{(k)}_{1/2} - 2(k-1)NH(\alpha_k)\frac{\partial\gamma_Q}{\partial\alpha_{k-1}}\alpha_{k-1}M^{(k-1)}_{1/2},$$

$$\mu\frac{dM^{(k-1)}_{1/2}}{d\mu} = -N(k-3+2k\gamma_Q)H'(\alpha_{k-1})\alpha_{k-1}M^{(k-1)}_{1/2} - 2kNH(\alpha_{k-1})\frac{\partial\gamma_Q}{\partial\alpha_{k-1}}\alpha_{k-1}M^{(k-1)}_{1/2} - 2kNH(\alpha_{k-1})\frac{\partial\gamma_Q}{\partial\alpha_k}\alpha_k M^{(k)}_{1/2},$$

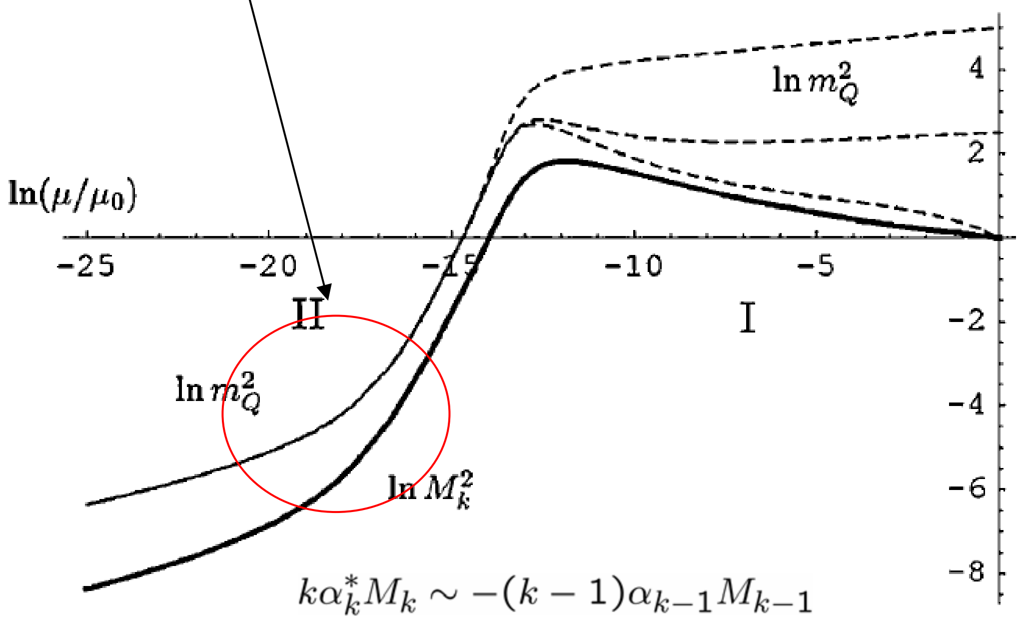
FIG. 2. RG running of the gaugino masses $M^{(k-1)}_{1/2}(\mu)$ and $M^{(k)}_{1/2}(\mu)$ with respect to $\ln(\mu/\mu_0)$. The gauge couplings are given at $\mu = \mu_0$ as $(\alpha_k, \alpha_{k-1}) = (0.0128, 0.04)$ and run along the renormalized trajectory. $k = 5$.

$$\gamma_Q = -N[k\alpha_k + (k-1)\alpha_{k-1}]$$

Soft scalar mass:

(1-loop anomalous dimension)

$$\frac{dm_Q^2}{d\ln(\mu)} \sim 0. \longrightarrow m_Q^2 \to \frac{1}{k(\alpha_k^*)^2}\alpha_{k-1}|M_{1/2}^{(k-1)}|^2$$



$$k\alpha_k^* M_k \sim -(k-1)\alpha_{k-1}M_{k-1}$$

FIG. 3. RG running behaviors of the scalar mass $\ln m_Q^2$ and the gaugino mass $2\ln M_{1/2}^{(k)}$ are shown by dotted lines and the bold line, respectively.

initial values are taken as $\ln m_Q^2 = 0,\ 2.5,\ 5.0$

$$\mu\frac{dm_Q^2}{d\mu} = -k\alpha_k(2|M_{1/2}^{(k)}|^2 + \Delta_k) - (k-1)\alpha_{k-1}(2|M_{1/2}^{(k-1)}|^2 + \Delta_{k-1}),$$

where

$$\Delta_k = H(\alpha_k)[3k|M_{1/2}^{(k)}|^2 - 2(k-1)m_Q^2],$$

$$\Delta_{k-1} = H(\alpha_{k-1})[3(k-1)|M_{1/2}^{(k-1)}|^2 - 2km_Q^2].$$

$$N\alpha \to \alpha,$$

$$\frac{(m_Q^2 + m_{\bar{Q}}^2)}{2} \to m_Q^2.$$

4. RG equations of soft masses in dual theory

RG equations of Gaugino masses M and trilinear holomorphic scalar coupling a $(\because W_{dual} \sim y\bar{q}Mq)$ around fixed point:

$$\frac{d}{d\ln(\mu)}\begin{pmatrix}\alpha^*_{k-2}M_{k-2}\\ -y^\dagger_* a\\ \alpha_{k-1}M_{k-1}\end{pmatrix} = \begin{pmatrix} \left.\frac{\partial\beta_{k-2}}{\partial\alpha_{k-2}}\right|_* & \left.\frac{\partial\beta_{k-2}}{\partial\alpha_y}\right|_* & \left.\frac{\partial\beta_{k-2}}{\partial\alpha_{k-1}}\right|_* \\ \left.\frac{\partial\beta_{\alpha_y}}{\partial\alpha_{k-2}}\right|_* & \left.\frac{\partial\beta_{\alpha_y}}{\partial\alpha_y}\right|_* & \left.\frac{\partial\beta_{\alpha_y}}{\partial\alpha_{k-1}}\right|_* \\ \left.\frac{\partial\beta_{k-1}}{\partial\alpha_{k-2}}\right|_* & \left.\frac{\partial\beta_{k-1}}{\partial\alpha_y}\right|_* & \left.\frac{\partial\beta_{k-1}}{\partial\alpha_{k-1}}\right|_* \end{pmatrix} \begin{pmatrix}\alpha^*_{k-2}M_{k-2}\\ -y^\dagger_* a\\ \alpha_{k-1}M_{k-1}\end{pmatrix}.$$

* stands for substitution of $g_{k-2} \approx g^*_{k-2},\ y \approx y_*,\ g_{k-1} \ll 1.$

Soft terms (M_{k-2}, a) shrink because of a infrared attractive nature of fixed point. But soft mass terms again converge to M_{k-1} who changes very slowly because M_{k-1} is in the equations.

$$dm_{soft}/d\ln(\mu) \sim 0. \longrightarrow \alpha^*_{k-2}M_{k-2} \sim -y^\dagger_* a \sim \alpha_{k-1}M_{k-1}.$$

Sum of scalar mass around fixed point:

$$(\alpha^*_{k-2}(m^2_q + m^2_{\bar{q}}),\ \alpha^*_y(m^2_q + m^2_{\bar{q}} + m^2_M),\ \alpha_{k-1}|M_{k-1}|^2)$$

satisfy the similar equations. Therefore

$$\alpha^*_y(m^2_q + m^2_{\bar{q}} + m^2_M) \sim \alpha^*_{k-2}(m^2_q + m^2_{\bar{q}}) \sim \alpha_{k-1}|M_{k-1}|^2.$$

In other words,

$$\alpha^*_y m^2_M \sim \alpha^*_{k-2}(m^2_q + m^2_{\bar{q}}) \sim \alpha_{k-1}|M_{k-1}|^2 \quad \text{for } \alpha^*_{k-2} \sim \alpha^*_y.$$

Comment on quartic coupling, bilinear coupling and matching:

$$W = h\,\det(Q_r Q_s), \quad W_{dual} \sim m\,\det(M_{rs})$$

$$\longrightarrow \quad \mathcal{L}^{(ori)}_{soft} \supset -a_h \det(\tilde{Q}_r \tilde{\bar{Q}}_s), \quad \mathcal{L}^{(dual)}_{soft} \supset -b\,\det(M_{rs}).$$

$A_h = a_h/h$ and $B = b/m$ would not change drastically for $h\mu \ll 1$, because both h and m do not have fixed point.

h: irrelevant but relevant near IR fixed point, m: relevant

(When h is irrelevant, it can be negligibly small as h~1/m'. m': heavy meson mass of previous step.)

For $h\mu \gg 1$, we treated m and b instead of h and a_h through a duality and match them by hand:

$$h(\Lambda_k)\Lambda_k = \frac{m(\Lambda_{k-2})}{\Lambda_{k-2}}, \; M_k(\Lambda_k) = M_{k-2}(\Lambda_{k-2}), \; A_h(\Lambda_k) = B(\Lambda_{k-2}).$$

$$\Lambda_k \equiv \Lambda_{k-2}$$

Λ_i: scale where theory becomes strong and approach to an IR fixed point.

For m >> μ >> B (and other soft terms), we can integrate out meson in an approximately supersymmetric manner. Then

$$W_{low} = -\frac{(y_* - a\theta^2)^2}{(m - b\theta^2)} \det_{r,s}(q_r \bar{q}_s)$$

$$= \tilde{h}' \det_{r,s}(q_r \bar{q}_s)$$

$$\tilde{h}' = -\frac{y_*^2}{m}[1 + (B - 2A_y)\theta^2] = (\tilde{h} - a_{\tilde{h}}\theta^2).$$

$$A_y = a/y$$

We have threshold effect (~ gauge mediation) from meson, too.

$$\Delta M_{k-1} \sim \alpha_{k-1} B, \;\; \Delta m_q^2 = \Delta m_{\bar{q}}^2 \sim \alpha_{k-1}^2 |B|^2.$$

Duality cascade will continue. (SU((k-1)N) becomes strong.)

After many times of duality cascade, that is, for μ $\gtrsim$ m ~ B ~ $\sqrt{m_M^2}$, we obtain mass of meson, which are adjoint (and singlet) for weakly interacting gauge theory;

$$|m|^2 + m_M^2 \pm |b|.$$

Off-diagonal part of scalar mass

We could find tachyonic mode and then weakly interacting gauge group may break by $\langle M \rangle \neq 0$.

ref. EWSB in MSSM

Then cascade would be terminated.

・ We may have another possibilities for the gauge symmetry breaking, i.e. the end of the cascade.

・ Structure of vacuum can depend on the model of the duality cascade. (Safely EWSB or CCB, UFB…)

5. Summary

What we did :

・ We study evolution of renormalization group of both supersymmetric terms and SUSY breaking terms in SU(kN)×SU((k-1)N) model of cascade under 1-loop anomalous dimension (=1-loop or 2-loop beta function).

What we found :

・ Almost SUSY breaking terms are suppressd and converge to weak coupling gaugino mass in the infrared regime.
(a kind of conformal sequestering of soft mass)

・ Because of SUSY breaking term (holomorphic mass term (B-term) here), we could find gauge symmetry breaking; the cascade could end.

Backup

We have another possibilities which we did not study.

Soft masses for singlet mesons M_0 may be driven to be negative because of the Yukawa couplings.

Similarly, the singlet meson fields M_0 may develop their VEVs depending on values of their various mass terms. Their VEVs induce mass terms of dual quarks. If such masses are large enough, the dual quarks would decouple and the flavor number would reduce to be outside of the conformal window. Then, the cascade could end. In addition, scalar components of q_r and $\bar{q}_s$ may develop their VEVs depending on values the A-terms and their soft scalar masses as well as other parameters in the scalar potential. Their VEVs break gauge symmetry and the cascade would end.

If the quartic A-term is comparable with SUSY breaking scalar masses m_Q, the origin of the scalar potential of Q would be unstable and similar symmetry breaking would happen. Such gauge symmetry breaking with reducing the flavor number may correspond to the symmetry breaking by VEVs of M with inducing dual quark masses.

And so on…

Beta function for scalar mass m_q^2 under 1-loop approximation

$$
\begin{aligned}
\frac{dm_q^2}{d\ln\mu} &= \left(\frac{\partial\gamma_q}{\partial\alpha_{k-2}}\tilde{\alpha}_{k-2} + \frac{\partial\gamma_q}{\partial\alpha_{k-1}}\tilde{\alpha}_{k-1} + \frac{\partial\gamma_q}{\partial\alpha_y}\tilde{\alpha}_y\right)\Bigg|_{\theta^2\bar{\theta}^2} \\
&= -(k-2)\alpha_{k-2}\left(2|M_{k-2}|^2 + \Delta'_{k-2}\right) - (k-1)\alpha_{k-1}\left(2|M_{k-1}|^2 + \Delta'_{k-1}\right) \\
&\quad +2(k-1)\alpha_y(\Sigma^2 + |A_y|^2) \\
\frac{dm_M^2}{d\ln\mu} &= \left(\frac{\partial\gamma_M}{\partial\alpha_{k-1}}\tilde{\alpha}_{k-1} + \frac{\partial\gamma_M}{\partial\alpha_y}\tilde{\alpha}_y\right)\Bigg|_{\theta^2\bar{\theta}^2} \\
&= -2(k-1)\alpha_{k-1}\left(2|M_{k-1}|^2 + \Delta'_{k-1}\right) + (k-2)\alpha_y(\Sigma^2 + |A_y|^2)
\end{aligned}
$$

$$
\begin{aligned}
\Delta'_{k-2} &= \frac{\alpha_{k-2}}{1-(k-2)\alpha_{k-2}}\left[3(k-2)|M_{k-2}|^2 - 2(k-1)m_q^2\right] \\
\Delta'_{k-1} &= \frac{\alpha_{k-1}}{1-(k-1)\alpha_{k-1}}\left[3(k-1)|M_{k-1}|^2 - 2(k-2)m_q^2 - 4m_M^2\right] \\
\Sigma^2 &= 2m_q^2 + m_M^2 \qquad (m_q^2 = m_{\tilde{q}}^2), \quad a = yA_y.
\end{aligned}
$$

$$
\begin{aligned}
\gamma_q &= -(k-2)\alpha_{k-2} - (k-1)\alpha_{k-1} + 2(k-1)\alpha_y \\
\gamma_M &= -2(k-1)\alpha_{k-1} + (k-2)\alpha_y
\end{aligned}
$$

Around fixed point B

$$
\gamma_q^* = -\frac{k-4}{2(k-1)}, \qquad \gamma_M^* = \frac{k-4}{k-1}
$$

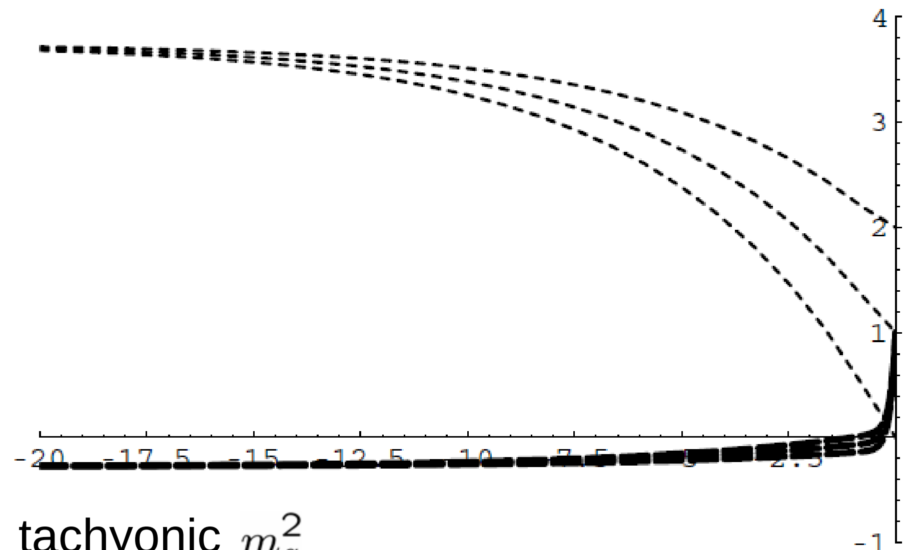
$$
\alpha_y^* = \frac{k-4}{(k-1)(k-2)}, \qquad \alpha_{k-2}^* = \frac{5k-6}{2(k-2)}\alpha_y^*
$$

$$
\begin{aligned}
\frac{dm_q^2}{d\ln\mu} &\simeq 2(k-1)(k-2)\alpha_{k-2}^* m_q^2 + 2(k-1)\alpha_y^*\Sigma^2 - 2(k-1)\alpha_{k-1}|M_{k-1}|^2 \\
\frac{dm_M^2}{d\ln\mu} &\simeq +(k-2)\alpha_y^*\Sigma^2 - 4(k-1)\alpha_{k-1}|M_{k-1}|^2
\end{aligned}
$$

- - - $m_q^2 = m_{\tilde{q}}^2$

.... m_M^2

$\alpha_{k-1}|M_{k-1}|^2 = 0.05 = \text{constant.}$



k=5
(We do not know for large k, large ‘tHooft coupling)

tachyonic m_q^2

Holomorphic quartic term on a surface of $\eta = \mu h << 1$

Ratio $A_h \equiv a_h / h$

$$\frac{dA_h}{d\ln(\mu)} = 4Nk\alpha_k M_k + 4N(k-1)\alpha_{k-1}M_{k-1} + \mathcal{O}(|\eta|^2)A_h$$

Excluding around fixed point B:

$$\frac{dA_h}{d\ln(\mu)} \sim 4Nk\alpha_k M_k > 0.$$

A_h can decrease.
(and could become negative.)

Around fixed point B: $k\alpha_k^* M_k \sim -(k-1)\alpha_{k-1}M_{k-1}$

$$\frac{dA_h}{d\ln(\mu)} \sim \mathcal{O}(|\eta|^2)A_h \sim 0.$$

Therefore magnitude of A_h would not change drastically if gaugino mass are not much larger than A_h at the initial condition.

Holomorphic quadratic term on a surface of $\hat{\eta} = m/\mu \ll 1$

Ratio $B \equiv b/m$ and $a = yA_y$

$$\frac{dB}{d\ln(\mu)} = 4N(k-1)\alpha_{k-1}M_{k-1} + N(k-2)\alpha_y A_y$$

Excluding around fixed point B:

$$\frac{dB}{d\ln(\mu)} \sim N(k-2)\alpha_y A_y > 0.$$

B can decrease.
(and could become negative.)

Around fixed point B:

$$\frac{dB}{d\ln(\mu)} \sim \alpha_{k-1}M_{k-1} \sim 0.$$

Therefore magnitude of B would not change drastically if gaugino mass and A_y are not much larger than B at the initial condition.

Illustrating model : Breaking of L-R symmetry by L-R Higgs

$U(3)\times USp(6)_L\times USp(6)_R\times U(1)$

$3\times$ $\tilde{Q}_L$: $(3, 6, 1, 0)$, $\tilde{Q}_R$: $(\bar{3}, 1, 6, 0)$, $\tilde{L}_L$: $(1, 6, 1, -1)$, $\tilde{L}_R$: $(1, 1, 6, 1)$

$$W = h\tilde{Q}_L\tilde{Q}_R\tilde{L}_L\tilde{L}_R.$$

$U(3)\times USp(2)_L\times USp(2)_R\times U(1)$

$3\times$ $\hat{Q}_L$: $(\bar{3}, 2, 1, 0)$, $\hat{Q}_R$: $(3, 1, 2, 0)$, L_L: $(1, 2, 1, 1)$, and L_R: $(1, 1, 2, -1)$.

$$W = \hat{h}\hat{Q}_L\hat{Q}_R L_L L_R.$$

$U(3)\times USp(2)_L\times USp(2)_R\times U(1)$

$3\times$ Q_L: $(3, 2, 1, 0)$, Q_R: $(\bar{3}, 1, 2, 0)$, L_L: $(1, 2, 1, 1)$, L_R: $(1, 1, 2, -1)$

$9\times$ H: $(1, 2, 2, 0)$ (Meson of above SU(3)) $W = y_Q Q_L Q_R H + y_L L_L L_R H + mHH.$

By hand