Theoretical aspects of fermion-flavour mixing

Andrea A. Almasy

Hamburg University

Outline

- history
- an explicit on-shell renormalization prescription
  - main idea
  - mass counterterm
  - mixing-matrix counterterm
- particular cases
  - quark mixing
  - lepton mixing
- quark-mixing renormalization effects on the determination of $|V_{ij}|$
- summary

History

- 1972 → renormalizability of the SM without quark-flavor mixing ['t Hooft and Veltman, 1972] (Nobel prize 1999)
- elements of mixing matrices appear as basic parameters in the bare Lagrangian → subject to renormalization, too
- 1975 → Cabibbo angle in the SM with two fermion generations [Marciano and Sirlin, 1975]
- 1990 → CKM matrix of the three-generation SM [Denner and Sack, 1990]

History

- 1972 $\rightarrow$ renormalizability of the SM without quark-flavor mixing ['t Hooft and Veltman, 1972] (Nobel prize 1999)
- elements of mixing matrices appear as basic parameters in the bare Lagrangian $\rightarrow$ subject to renormalization, too
- 1975 $\rightarrow$ Cabibbo angle in the SM with two fermion generations [Marciano and Sirlin, 1975]
- 1990 $\rightarrow$ CKM matrix of the three-generation SM [Denner and Sack, 1990]

  $\rightarrow$ later shown that it leads to gauge-dependent results

History

- 1990 – present $\rightarrow$ CKM matrix renormalization prescriptions
  - subtraction point $q^2 = 0$ [Gambino, Grassi and Madricardo, 1999]
  - pinch technique [Yamada, 2001]
  - reference theory with no mixing [Barroso, Brücher and Santos, 2000; Diener and Kniehl, 2001; Zhou, 2003; Denner, Kraus and Roth, 2004]
  - self-mass [Kniehl and Sirlin, 2006; 2009]
- generalization to extensions of the SM $\rightarrow$ lepton mixing
  - reference theory with no mixing [Diener and Kniehl, 2001]
  - self-mass [Almasy, Kniehl and Sirlin, 2009]

History

- 1990 – present $\rightarrow$ CKM matrix renormalization prescriptions
  - subtraction point $q^2 = 0$ [Gambino, Grassi and Madricardo, 1999]
  - pinch technique [Yamada, 2001]
  - reference theory with no mixing [Barroso, Brücher and Santos, 2000; Diener and Kniehl, 2001; Zhou, 2003; Denner, Kraus and Roth, 2004]
  - self-mass [Kniehl and Sirlin, 2006; 2009]
- generalization to extensions of the SM $\rightarrow$ lepton mixing
  - reference theory with no mixing [Diener and Kniehl, 2001]
  - self-mass [Almasy, Kniehl and Sirlin, 2009]

Main idea

- generalization of Feynman's approach in QED

$$\Delta\mathcal{M}^{\mathrm{leg}} = \bar{u}(p)\Sigma(\not{p})\frac{1}{\not{p}-m}$$

with $\Sigma(\not{p}) = \underbrace{A(p^2) + B(p^2)}\,(\not{p}-m) + \underbrace{\Sigma_{\mathrm{fin}}(\not{p})}$, the self-energy

divergent constants ↙ ↘ $\propto\ (\not{p}-m)^2$ *in the vicinity of* $\not{p}=m$

- A, referred to as sm, has a pole at $\not{p}=m$ and is gauge-parameter independent $\to$ cancelled by the mass counterterm
- B, referred to as wfr, is regular at $\not{p}=m$ but gauge-parameter dependent $\to$ combined with the proper vertex diagrams leading to a gauge-parameter-independent result

Main idea

- generalization of Feynman's approach in QED $\rightarrow$ mixing

$$\Delta\mathcal{M}_{ij}^{\text{leg}} = \bar{u}_i(p)\Sigma_{ij}(\not{p})\frac{1}{\not{p}-m_j}$$

$$\begin{aligned}\Sigma_{ij}(\not{p}) &= A_{ij}(p^2) \\ &+ (\not{p}-m_i)B_{ij}^L(p^2) + B_{ij}^R(p^2)(\not{p}-m_j) \\ &+ (\not{p}-m_i)\Sigma_{ij}^{\text{fin}}(p^2)(\not{p}-m_j)\end{aligned}$$

- A, referred to as sm, has a pole at $\not{p}=m$ and is gauge-parameter independent $\rightarrow$ cancelled by the mass counterterm
- B, referred to as wfr, is regular at $\not{p}=m$ but gauge-parameter dependent $\rightarrow$ combined with the proper vertex diagrams leading to a gauge-parameter-independent result

Main idea

- generalization of Feynman's approach in QED → mixing

$$\Delta\mathcal{M}_{ij}^{\rm leg} = \bar{u}_i(p)\Sigma_{ij}(\not p)\frac{1}{\not p - m_j}$$

$$\begin{aligned} \Sigma_{ij}(\not p) &= A_{ij}(p^2) \rightsquigarrow \text{sm} \\ &+ (\not p - m_i)B_{ij}^L(p^2) + B_{ij}^R(p^2)(\not p - m_j) \rightsquigarrow \text{wfr} \\ &+ (\not p - m_i)\Sigma_{ij}^{\rm fin}(p^2)(\not p - m_j) \rightsquigarrow 0 \end{aligned}$$

- A, referred to as sm, has a pole at $\not p = m$ and is gauge-parameter independent → cancelled by the mass counterterm
- B, referred to as wfr, is regular at $\not p = m$ but gauge-parameter dependent → combined with the proper vertex diagrams leading to a gauge-parameter-independent result

Mass counterterm

- bare mass term: $-\overline{\psi}'_R m'_0 \psi'_L + \mathrm{H.c.}$
- decompose $m'_0 = m' + \delta m'$
- bi-unitary transformation of $\overline{\psi}', \psi' \to m'$ is diagonal
- new framework

$$-\overline{\psi}(m+\delta m^{(-)}P_L+\delta m^{(+)}P_R)\psi = -\overline{\psi}_R(m+\delta m^{(-)})\psi_L-\overline{\psi}_L(m+\delta m^{(+)})\psi_R$$

- m is real, diagonal and positive
- $\delta m^{(\pm)}$ are arbitrary non-diagonal matrices

$$\delta m^{(+)} = \delta m^{(-)\dagger}$$

- adjust $\delta m^{(\pm)}$ to cancel the sm contributions to $\Delta\mathcal{M}^{\mathrm{leg}}$

Mixing-matrix counterterm

- complete mass matrix

$$M = m + \delta m^{(-)} P_L + \delta m^{(+)} P_R$$

- bi-unitary transformation

$$\psi_{L,R} = U_{L,R} \hat{\psi}_{L,R}$$

Mixing-matrix counterterm

- complete mass matrix

$$M = m + \delta m^{(-)} P_L + \delta m^{(+)} P_R$$

- bi-unitary transformation

$$\psi_{L,R} = U_{L,R} \hat{\psi}_{L,R} \approx (1 + i h_{L,R}) \hat{\psi}_{L,R}$$

$\searrow$ *at one loop*

Mixing-matrix counterterm

- complete mass matrix

$$M = m + \delta m^{(-)} P_L + \delta m^{(+)} P_R$$

- bi-unitary transformation

$$\psi_{L,R} = U_{L,R} \hat{\psi}_{L,R} \approx (1 + i h_{L,R}) \hat{\psi}_{L,R}$$

↘ *at one loop*

- choose $h_{L,R}$ such that $\hat{M}$ is diagonal

$$i(h_{L,R})_{ij} = -\frac{m_i \delta m_{ij}^{(\mp)} + \delta m_{ij}^{(\pm)} m_j}{m_i^2 - m_j^2}, \qquad (h_{L,R})_{ii} = 0$$

Mixing-matrix counterterm

- $V f_i \overline{f}_j$ bare interaction

$$\mathcal{L}_{V f_i \overline{f}_j} \propto \overline{\psi}_L^{f_i} K_0 \gamma^\lambda \psi_L^{f_j} V_\lambda + H.c.$$

- new framework

$$\mathcal{L}_{W f_i \overline{f}_j} \propto \overline{\hat{\psi}}_L^{f_i} (K + \delta K) \gamma^\lambda \hat{\psi}_L^{f_j} V_\lambda + H.c.$$

with $\delta K = i(K h_L^{f_j} - h_L^{f_i} K)$

- both $K_0 = K + \delta K$ and K are explicitly gauge independent and preserve the basic properties of the theory
- K is finite $\Rightarrow$ identified with the renormalized mixing matrix
- δK identified with the mixing-matrix counterterm

Quark mixing

- CKM matrix counterterm [Kniehl and Sirlin, 2006]

$$\delta V = i \left(V h_L^D - h_L^U V \right)$$

- both $V_0 = V + \delta V$ and V satisfy the unitarity condition and are explicitly gauge independent
- the sm corrections are fully cancelled in the amplitude associated with V_{ud}, the most accurately measured CKM parameter
- the residual finite contributions to other channels are very small

Quark mixing. Alternative approach

- on covariant grounds, the quark self-energy $\Sigma_{ij}(\not{p})$ has the form

$$\Sigma_{ij}(\not{p}) = \not{p} P_L \Sigma_{ij}^L(p^2) + \not{p} P_R \Sigma_{ij}^R(p^2) + (m_i P_L + m_j P_R)\Sigma_{ij}^S(p^2)$$

- proposed mass counterterms [Kniehl and Sirlin, 2009]

$$\delta m_{ij}^{(-)} = \frac{m_i}{2}\Sigma_{ij}^L(m_j^2) + \frac{m_j}{2}\Sigma_{ij}^R(m_j^2) + m_i \Sigma_{ij}^S(m_j^2),$$
$$\delta m_{ij}^{(+)} = \frac{m_j}{2}\Sigma_{ij}^L(m_i^2) + \frac{m_i}{2}\Sigma_{ij}^R(m_i^2) + m_j \Sigma_{ij}^S(m_i^2).$$

- important properties
  - gauge independence

Quark mixing. Alternative approach

- on covariant grounds, the quark self-energy $\Sigma_{ij}(\not{p})$ has the form

$$\Sigma_{ij}(\not{p}) = \not{p} P_L \Sigma^L_{ij}(p^2) + \not{p} P_R \Sigma^R_{ij}(p^2) + (m_i P_L + m_j P_R)\Sigma^S_{ij}(p^2)$$

- proposed mass counterterms [Kniehl and Sirlin, 2009]

$$\delta m^{(-)}_{ij} = \frac{m_i}{2}\Sigma^L_{ij}(m_j^2) + \frac{m_j}{2}\Sigma^R_{ij}(m_j^2) + m_i \Sigma^S_{ij}(m_j^2),$$

$$\delta m^{(+)}_{ij} = \frac{m_j}{2}\Sigma^L_{ij}(m_i^2) + \frac{m_i}{2}\Sigma^R_{ij}(m_i^2) + m_j \Sigma^S_{ij}(m_i^2).$$

- important properties
  - gauge independence →Proof: Nielsen identities employed to show that [Espriu, Manzano and Talavera, 2002]

$$\partial_\xi \left(m_i m_j \Sigma^L_{ij}(p^2) + p^2 \Sigma^R_{ij}(p^2) + 2 m_i m_j \Sigma^S_{ij}(p^2) \right) = 0$$

Quark mixing. Alternative approach

- on covariant grounds, the quark self-energy $\Sigma_{ij}(\not{p})$ has the form

$$\Sigma_{ij}(\not{p}) = \not{p} P_L \Sigma^L_{ij}(p^2) + \not{p} P_R \Sigma^R_{ij}(p^2) + (m_i P_L + m_j P_R)\Sigma^S_{ij}(p^2)$$

- proposed mass counterterms [Kniehl and Sirlin, 2009]

$$\delta m^{(-)}_{ij} = \frac{m_i}{2}\Sigma^L_{ij}(m_j^2) + \frac{m_j}{2}\Sigma^R_{ij}(m_j^2) + m_i \Sigma^S_{ij}(m_j^2),$$
$$\delta m^{(+)}_{ij} = \frac{m_j}{2}\Sigma^L_{ij}(m_i^2) + \frac{m_i}{2}\Sigma^R_{ij}(m_i^2) + m_j \Sigma^S_{ij}(m_i^2).$$

- important properties
  - gauge independence →Proof: Nielsen identities employed to show that [Espriu, Manzano and Talavera, 2002]

$$\partial_\xi \left(m_i m_j \Sigma^L_{ij}(m_i^2) + m_i^2 \Sigma^R_{ij}(m_i^2) + 2 m_i m_j \Sigma^S_{ij}(m_i^2)\right) = 0$$

Quark mixing. Alternative approach

- on covariant grounds, the quark self-energy $\Sigma_{ij}(\not{p})$ has the form

$$\Sigma_{ij}(\not{p}) = \not{p} P_L \Sigma_{ij}^L(p^2) + \not{p} P_R \Sigma_{ij}^R(p^2) + (m_i P_L + m_j P_R)\Sigma_{ij}^S(p^2)$$

- proposed mass counterterms [Kniehl and Sirlin, 2009]

$$\delta m_{ij}^{(-)} = \frac{m_i}{2}\Sigma_{ij}^L(m_j^2) + \frac{m_j}{2}\Sigma_{ij}^R(m_j^2) + m_i \Sigma_{ij}^S(m_j^2),$$
$$\delta m_{ij}^{(+)} = \frac{m_j}{2}\Sigma_{ij}^L(m_i^2) + \frac{m_i}{2}\Sigma_{ij}^R(m_i^2) + m_j \Sigma_{ij}^S(m_i^2).$$

- important properties
  - gauge independence
  - automatically satisfy the hermiticity constraint

Quark mixing. Alternative approach

- on covariant grounds, the quark self-energy $\Sigma_{ij}(\not p)$ has the form

$$\Sigma_{ij}(\not p) = \not p P_L \Sigma^L_{ij}(p^2) + \not p P_R \Sigma^R_{ij}(p^2) + (m_i P_L + m_j P_R)\Sigma^S_{ij}(p^2)$$

- proposed mass counterterms [Kniehl and Sirlin, 2009]

$$\delta m^{(-)}_{ij} = \frac{m_i}{2}\Sigma^L_{ij}(m_j^2) + \frac{m_j}{2}\Sigma^R_{ij}(m_j^2) + m_i \Sigma^S_{ij}(m_j^2),$$
$$\delta m^{(+)}_{ij} = \frac{m_j}{2}\Sigma^L_{ij}(m_i^2) + \frac{m_i}{2}\Sigma^R_{ij}(m_i^2) + m_j \Sigma^S_{ij}(m_i^2).$$

- important properties
  - gauge independence
  - automatically satisfy the hermiticity constraint
  - expressed in terms of the invariant self-energy functions $\Rightarrow$ useful for practical applications

Lepton mixing

- SM with right-handed neutrinos (heavy Majorana neutrinos)
  - N_G fermion generations, $(\nu'^0_{L,i}, l'^0_{L,i})$ and $l'^0_{R,i}$
  - N_R right-handed neutrinos, $\nu'^0_{R,i}$
- mixing appears in
  - charged-current interaction $\rightarrow B^0_{ia} = \sum_k V^{0,l}_{ik} U^{0,\nu *}_{ka}$
  - neutral-current interaction $\rightarrow C^0_{ab} = \sum_c U^{0,\nu T}_{ac} U^{0,\nu *}_{cb}$
- properties of the mixing matrices

$$\sum_c B^0_{ic} B^{0*}_{jc} = \delta_{ij}, \quad \sum_i B^{0*}_{ia} B^0_{ib} = C^0_{ab}$$

$$\sum_c B^0_{ic} C^0_{ca} = B^0_{ia}, \quad \sum_c C^0_{ac} C^0_{cb} = C^0_{ab} = C^{0\dagger}_{ab}$$

Lepton mixing

- charged-lepton mass counterterm:
  $\rightarrow$ identical to that of quarks, up to particle content
- neutrino mass counterterm:
  $\rightarrow$ changes due to Majorana constraint $\nu = \nu^C$
  - $\delta m^{\nu(+)} = \delta m^{\nu(+)T} = \delta m^{\nu(-)*} = \delta m^{\nu(-)\dagger}$
  - $U^\nu = 1 + ih^\nu \Rightarrow \hat{M}^\nu$ diagonal
- mixing counterterms [Almasy, Kniehl, and Sirlin, 2009]

$$\delta B = i\left(Bh^\nu - h^l_L B\right) \qquad \text{and} \qquad \delta C = i\left(Ch^\nu - h^\nu C\right)$$

- once δB is fixed, δC is fixed as well
- both, the bare and renormalized mixing matrices, are gauge independent and preserve the basic properties of the theory

Effects on the determination of $|V_{ij}|$

- Partial hadronic decay widths of the W boson at one-loop level

  [Almasy, Kniehl and Sirlin, 2008]

Partial width	[1]	[2]	[3]	[4]	[5]	$\overline{\mathrm{MS}}^{\dagger}$	$\delta V_{ij} = 0^{*}$
$\Gamma(W \to ud)$	0.6697016	0.6697016	0.6697016	0.6697016	0.6697016	0.6696999	0.6697012
$\Gamma(W \to us) \times 10$	0.3594604	0.3594604	0.3594604	0.3594604	0.3594604	0.3594804	0.3590518
$\Gamma(W \to ub) \times 10^4$	0.0934579	0.0930919	0.0934544	0.0934578	0.0934580	0.0904068	0.0906568
$\Gamma(W \to cd) \times 10$	0.3589746	0.3589746	0.3589746	0.3589746	0.3589746	0.3589738	0.3556135
$\Gamma(W \to cs)$	0.6684818	0.6684818	0.6684819	0.6684818	0.6684818	0.6684267	0.6614634
$\Gamma(W \to cb) \times 10^2$	0.1211309	0.1211315	0.1211263	0.1211318	0.1211315	0.1266316	0.1196919

[1] Denner and Sack, 1990.

[2] Gambino, Grassi and Madricardo, 1999.

[3] Diener and Kniehl, 2001.

[4] Kniehl and Sirlin, 2006.

[5] Kniehl and Sirlin, 2009.

† the ’t Hooft mass scale was chosen to be $\mu = m_W$

* $V_{ij} = \delta_{ij}$ considered in the calculation of the one-loop corrections.

Effects on the determination of $|V_{ij}|$

- partial widths at one loop level

$$\Gamma(W^+ \to q_i \bar{q}_j) \propto |V'^{\alpha}_{ij}|^2 (1 + \delta^{\alpha}_{ij}) = |V_{ij}|^2 (1 + \delta^0_{ij})$$

- it implies

$$R^{\alpha}_{ij} = \frac{|V'^{\alpha}_{ij}|^2}{|V_{ij}|^2} = \frac{1 + \delta^0_{ij}}{1 + \delta^{\alpha}_{ij}}$$

- permits the evaluation of the relative shifts in $|V_{ij}|^2$

$$\Delta^{\alpha}_{ij} = \frac{|V'^{\alpha}_{ij}|^2 - |V_{ij}|^2}{|V_{ij}|^2} = R^{\alpha}_{ij} - 1$$

Effects on the determination of $|V_{ij}|$

- Relative shifts Δ_{ij}^{α} (in %) in the central values of $|V_{ij}|^2$ (Particle Data Group, 2008) induced by quark-mixing renormalization effects. [Almasy, Kniehl and Sirlin, 2008]

Δ_{ij}^{α}(%) \ α	[1]	[2]	[3]	[4]	[5]	$\overline{\mathrm{MS}}^{\dagger}$	$\|V_{ij}\|^2$
$\Delta_{ud}^{\alpha} \times 10^5$	-5.29	-5.29	-5.26	-5.29	-5.29	20.00	0.9490462
Δ_{us}^{α}	-0.114	-0.114	-0.114	-0.114	-0.114	-0.119	0.5094049 $\times 10^{-1}$
Δ_{ub}^{α}	-3.00	-2.62	-2.99	-3.00	-3.00	0.277	0.1288810 $\times 10^{-4}$
Δ_{cd}^{α}	-0.936	-0.936	-0.936	-0.936	-0.936	-0.936	0.5089536 $\times 10^{-1}$
Δ_{cs}^{α}	-1.05	-1.05	-1.05	-1.05	-1.05	-1.04	0.9473908
Δ_{cb}^{α}	-1.19	-1.19	-1.18	-1.19	-1.19	-5.48	0.1722250 $\times 10^{-2}$

[1] Denner and Sack, 1990.

[2] Gambino, Grassi and Madricardo, 1999.

[3] Diener and Kniehl, 2001.

[4] Kniehl and Sirlin, 2006.

[5] Kniehl and Sirlin, 2009.

† the ’t Hooft mass scale was chosen to be $\mu = m_W$.

Summary

- review the status of fermion-flavour mixing renormalization
- explicit on-shell renormalization prescription
  - separate external leg corrections into sm and wfr contributions
  - choose mass counterterm to cancel, as much as possible, the sm contribution
  - diagonalize the complete mass matrix
- particular cases
  - quark mixing
  - lepton mixing
- quark-mixing renormalization effects on the determination of the CKM elements $|V_{ij}|$

The end.