# Reconstruction of Quark Mass Matrices with Weak Basis Texture Zeroes from Experimental Input

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**In Collaboration with**

**G.C. Branco, R. González Felipe, Phys. Lett. B 477 (2000) 147**
**G.C. Branco, R. González Felipe, Phys. Lett. B 483 (2000) 87**
**G.C. Branco, M.N. Rebelo, P. Roy, Phys. Rev. D 77 (2008) 053011 [Lepton Sector]**
**G.C. Branco, R. González Felipe, H. Serôdio, Phys. Lett. B 670 (2009) 340 [Lepton Sector]**
**C. Simões, Phys. Rev. D 79 (2009) 073006**

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# Talk's Outline

- Quark Masses and Mixings
- Texture Zeroes and Weak Basis Tranformations
- Power Structures and Flavour Symmetries
- Conclusions

# Standard Model Puzzles $SU(3)_C \times SU(2)_L \times U(1)_Y$

- **Naturalness Problems**
  - Strong CP problem: $d_n \Rightarrow \theta \lesssim 10^{-10}$
  - Higgs mass corrections: $\langle \phi^0 \rangle \sim v \sim |\mu|\,, \quad \mu_R^2 = \mu^2 + \alpha \Lambda^2 \sim m_W^2$
  - Cosmological constant: $V_\Phi(v) \neq 0$ (orders of magnitude $\sim 55$)

**A parameter in any theory remains small, if the symmetry is enhanced by setting that parameter to zero.**

**G. 't Hooft**

- **Mass and gauge coupling hierarchy**
  - Too many parameters (3 generations)
  - $g \sim g' < g_s$, $M_1^{q,l} \ll M_2^{q,l} \ll M_3^{q,l}$
  - $|V_{CKM}| \simeq 1$, the origin of the $\delta_{KM}$ CP phase
  - **Neutrinos** do have tiny masses $m_\nu$ and $|U_{PMNS}| \simeq U_{HPS}$
  - Electric charge and hypercharge quantisation unexplained: $|Q_p + Q_e| < 10^{-21}$

- **Gravity is not included!**

# Quark Sector

[Particle Data Group]

- Quark Mass Matrices
  - $m_u = U_{uL} \cdot \text{diag}(m_u,\, m_c,\, m_t) \cdot U^\dagger_{uR}$
  - $m_d = U_{dL} \cdot \text{diag}(m_d,\, m_s,\, m_b) \cdot U^\dagger_{dR}$
  - Unitary CKM matrix $V \equiv U^\dagger_{uL} U_{dL}$

- Running Masses at $M_Z$

$$m_u(M_Z) = 1.4^{+0.6}_{-0.5}\,\text{MeV}$$
$$m_d(M_Z) = 2.8 \pm 0.7\,\text{MeV}$$
$$m_s(M_Z) = 60^{+15}_{-19}\,\text{MeV}$$
$$m_c(M_Z) = 0.64^{+0.07}_{-0.09}\,\text{GeV}$$
$$m_b(M_Z) = 2.89^{+0.17}_{-0.08}\,\text{GeV}$$
$$m_t(M_Z) = 170.1 \pm 2.3\,\text{GeV}$$

- Wolfenstein Parametrisation

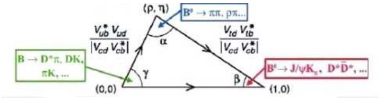
$$V \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Allowed range (fit)

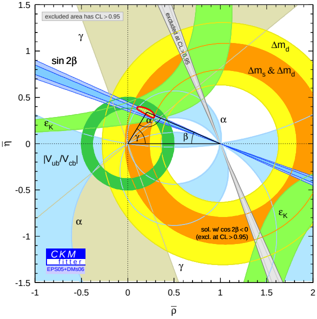
$$V = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

- $6+4$ Physical Parameters

## ➤ The Unitarity Triangle




$$V_{ud}\, V_{ub}^{*} + V_{cd}\, V_{cb}^{*} + V_{td}\, V_{tb}^{*} = 0$$




## ➤ CP Violation Quantities

$$\alpha = \arg(-V_{td}\, V_{tb}^{*}\, V_{ud}^{*}\, V_{ub})$$
$$\beta = \arg(-V_{cd}\, V_{cb}^{*}\, V_{td}^{*}\, V_{tb})$$
$$\gamma = \arg(-V_{ud}\, V_{ub}^{*}\, V_{cd}^{*}\, V_{cb})$$
$$J = \mathrm{Im}(V_{us}\, V_{ub}^{*}\, V_{cs}^{*}\, V_{cb})$$

## ➤ CKM Reconstruction

$$(V_{us},\ V_{ub},\ V_{cb},\ \sin 2\beta)$$
$$(V_{us},\ V_{ub},\ V_{cb},\ \sin \gamma)$$

## ➤ Experimental Values

$\alpha = \left(88^{+6}_{-5}\right)^{\circ}$ $\sin 2\beta = 0.681 \pm 0.025$ $\gamma = \left(77^{+30}_{-32}\right)^{\circ}$ $J = (3.05 \pm 0.19)\, 10^{-5}$

# Weak Basis Transformations

- General Transf. Leaving Charged Currents Invariant

  - **Quarks**

$$M_u \longrightarrow M_u' = W_q^\dagger \, M_u \, W_{uR}$$
$$M_d \longrightarrow M_d' = W_q^\dagger \, M_d \, W_{dR}$$

  - **Leptons**

$$m_\ell \longrightarrow m_\ell' = W_\ell^\dagger \, m_\ell \, W_{eR}$$
$$m_\nu \longrightarrow m_\nu' = W_\ell^T \, m_\nu \, W_\ell$$

- In the case of Hermitian Mass Matrices

$$W_{uR} = W_q \,, \quad W_{uR} = W_q \quad \text{and} \quad W_{eR} = W_\ell$$

# Texture Zeroes in the Fermion Mass Matrices

[Ramond,Robert,Ross
Frampton,Glashow,Marfatia
Nishiura,Matsuda,Fukuyama
Branco,D.E.C.,González Felipe
Fritzsch,Z. z. Xing]

- The relevance of Weak Basis Transformations
- Reduce Free Parameters (predictions)
- Horizontal Flavour Symmetries
  - Discrete symmetries
  - Extra Space-like Dimensions (*e.g.* fermion localisation)
  - Froggatt-Nielsen Mechanism (*e.g.* $U(1)$ horizontal )

$$M_{ij} \sim \left(\frac{v}{\Lambda}\right)^{Q_i+Q_j}$$

- Texture Zeroes
  - Can be some zeroes due to Weak Basis transformations?
  - Keeping $M_i$ Hermitian implies $W_R = W_L$
  - Such zeroes have no physical content

# Weak Basis Zeroes in the Quark Sector

[Branco,D.E.C.,González Felipe]

- Creating $(m'_u)_{11} = (m'_d)_{11} = 0$

$$\begin{cases} m_u\,|W_{11}|^2 + m_c\,|W_{21}|^2 + m_t\,|W_{31}|^2 = 0 \\ m_d\,|X_{11}|^2 + m_s\,|X_{21}|^2 + m_b\,|X_{31}|^2 = 0 \end{cases}$$

$$\begin{aligned} |X_{i1}|^2 \equiv\, & |V_{1i}|^2\,|W_{11}|^2 + |V_{2i}|^2\,|W_{21}|^2 + |V_{3i}|^2\,|W_{31}|^2 \\ & + 2Re\,(V^*_{1i} W_{11}\, V_{2i} W^*_{21}) + 2Re\,(V^*_{1i} W_{11}\, V_{3i} W^*_{31}) \\ & + 2Re\,(V^*_{2i} W_{21}\, V_{3i} W^*_{31}) \end{aligned}$$

- The sign choice of fermions are required in order to have solution

# Extra Weak Basis Zeroes

- Additional Weak Basis Zeroes

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -e^{i\varphi}\sin\theta \\ 0 & e^{-i\varphi}\sin\theta & \cos\theta \end{pmatrix}$$

- One possibility $(M_d)_{13} = 0$:

$$\tan\theta = \left|\frac{(M_d)_{13}}{(M_d)_{12}}\right|, \quad \varphi = \arg(M_d)_{13} - \arg(M_d)_{12}$$

- Other possibilities:

$$(m_d)_{22} = 0 \qquad \text{or} \qquad (m_d)_{23} = 0$$

- The Zero Position Can Move

  6 Permutation ($S_3$) matrices $\begin{cases} M'_u = P^{\mathsf{T}}\, M_u\, P \\ M'_d = P^{\mathsf{T}}\, M_d\, P \end{cases}$

- 3 Zeroes Without Physical Content

# Weak Basis Equivalent Classes

**Textures for $M_d$**

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$$

$$\begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix} \quad \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

$$\begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

## Textures for $M_u$

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$$

$$\begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix} \quad \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

$$\begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix} \quad \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix} \quad \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

- Can we have Weak Basis with more zeroes?

# Four-zero Parallel Ansätze

- No More Weak Parallel Zeroes

$$M_u = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} \qquad M_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$$

  - Not a Weak Basis!!!
  - $4+4+2=10$ parameters (Equal number of Physical Observables)
  - The Fourth Zero has really Physical Implications

- Nearest Neighbour Interaction (NNI)

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A'_u & 0 & B_u \\ 0 & B'_u & C_u \end{pmatrix} \quad M_d = \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A_d & 0 \\ A'_d & 0 & B_d \\ 0 & B'_d & C_d \end{pmatrix}$$

  - Non Hermitian Weak basis having 12 real parameters
  - Zeroes in NNI has no Physical Implications

# Quark Mass Matrices at $M_Z$

$$|M_u| = \begin{pmatrix} 0 & 0.0214 - 10.7 & 0.0137 - 2.58 \\ 0.0214 - 10.7 & 0.00358 - 172 & 0.00362 - 86.5 \\ 0.0137 - 2.58 & 0.00362 - 86.5 & 8.87 \times 10^{-8} - 172 \end{pmatrix}$$

$$|M_d| = \begin{pmatrix} 0 & 0.00959 - 0.322 & 0 \\ 0.00959 - 0.322 & 0.000146 - 3.05 & 0.0452 - 1.56 \\ 0 & 0.0452 - 1.56 & 0.00270 - 3.05 \end{pmatrix}$$

- Four Zero Textures seem not viable [for quarks]

# Weak Basis Zeroes at GUT Scale

- Standard Model

$$h_u(M_Z) = \frac{M_u(M_Z)}{v} \qquad h_d(M_Z) = \frac{M_d(M_Z)}{v}$$

- DHM (SM + an extra Higgs doublet)

$$h_u(M_Z) = \frac{M_u(M_Z)}{v \sin\beta} \qquad h_d(M_Z) = \frac{M_d(M_Z)}{v \cos\beta}$$

- MSSM (Minimal Supersymmetric SM in $\overline{DR}$ scheme)
  - 1st Step from $M_Z$ to $M_S \approx 1\text{TeV}$

$$h_u(M_Z) = \frac{M_u(M_Z)}{v} \qquad h_d(M_Z) = \frac{M_d(M_Z)}{v}$$

  - 2nd Step from $M_S$ to GUT scale

$$Y_u(M_S) = \frac{h_u(M_S)}{\sin\beta} \qquad Y_d(M_S) = \frac{h_d(M_S)}{\cos\beta}$$

- Power Structures $\lambda \approx 0.22$ [Cabbibo angle]

# Yukawa Coupling Matrices at GUT scale

## Standard Model

$\Lambda = 10^{14}$ GeV

Yukawa Couping ratios at GUT scale

$$y_t : y_c : y_u \approx 1 : \lambda^4 : \lambda^8$$

$$y_b : y_s : y_d \approx 1 : \lambda^{2-3} : \lambda^{4-5}$$

| | $h_{d13} = 0$ | $h_{d22} = 0$ | $h_{d23} = 0$ |
|---|---|---|---|
| $\lvert h_u \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.4-5.7} & \lambda^{3.1-6.4} \\ \lambda^{2.4-5.7} & \lambda^{0.5-3.6} & \lambda^{0.9-6.5} \\ \lambda^{3.1-6.4} & \lambda^{0.9-6.5} & \lambda^{0.5-13.4} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.6-8.8} & \lambda^{2.4-5.0} \\ \lambda^{3.6-8.8} & \lambda^{2.5-4.8} & \lambda^{1.5-2.6} \\ \lambda^{2.4-5.0} & \lambda^{1.5-2.6} & \lambda^{0.5} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.2-8.9} & \lambda^{2.4-5.0} \\ \lambda^{4.2-8.9} & \lambda^{3.9-7.2} & \lambda^{2.5-3.1} \\ \lambda^{2.4-5.0} & \lambda^{2.5-3.1} & \lambda^{0.5} \end{pmatrix}$ |
| $\lvert h_d \rvert$ | $\begin{pmatrix} 0 & \lambda^{4.8-7.0} & 0 \\ \lambda^{4.8-7.0} & \lambda^{3.3-7.2} & \lambda^{3.7-5.4} \\ 0 & \lambda^{3.7-5.4} & \lambda^{3.3-7.7} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{5.8-8.0} & \lambda^{4.8-8.4} \\ \lambda^{5.8-8.0} & 0 & \lambda^{4.5-5.1} \\ \lambda^{4.8-8.4} & \lambda^{4.5-5.1} & \lambda^{3.3} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{6.2-7.7} & \lambda^{4.8-8.2} \\ \lambda^{6.2-7.7} & \lambda^{5.6-6.9} & 0 \\ \lambda^{4.8-8.2} & 0 & \lambda^{3.3} \end{pmatrix}$ |

## Standard Model + an Extra Higgs Doublet

$\Lambda = 10^{14}$ GeV

| $\tan\beta$ | | $h_{d13} = 0$ | $h_{d22} = 0$ | $h_{d23} = 0$ |
|---|---|---|---|---|
| 10 | $\lvert h_u \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.4-6.1} & \lambda^{3.3-6.6} \\ \lambda^{2.4-6.1} & \lambda^{0.5-4.9} & \lambda^{0.9-7.1} \\ \lambda^{3.3-6.6} & \lambda^{0.9-7.1} & \lambda^{0.5-13.1} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.6-8.4} & \lambda^{2.4-4.9} \\ \lambda^{3.6-8.4} & \lambda^{2.6-5.1} & \lambda^{1.6-2.7} \\ \lambda^{2.4-4.9} & \lambda^{1.6-2.7} & \lambda^{0.5} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.3-9.5} & \lambda^{2.4-4.8} \\ \lambda^{4.3-9.5} & \lambda^{3.9-6.4} & \lambda^{2.6-3.3} \\ \lambda^{2.4-4.8} & \lambda^{2.6-3.3} & \lambda^{0.5} \end{pmatrix}$ |
| | $\lvert h_d \rvert$ | $\begin{pmatrix} 0 & \lambda^{3.4-5.6} & 0 \\ \lambda^{3.4-5.6} & \lambda^{1.8-7.8} & \lambda^{2.2-4.0} \\ 0 & \lambda^{2.2-4.0} & \lambda^{1.8-6.4} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.4-6.6} & \lambda^{3.3-8.5} \\ \lambda^{4.4-6.6} & 0 & \lambda^{3.0-3.8} \\ \lambda^{3.3-8.5} & \lambda^{3.0-3.8} & \lambda^{1.8-1.9} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.8-6.3} & \lambda^{3.4-7.5} \\ \lambda^{4.8-6.3} & \lambda^{4.3-5.7} & 0 \\ \lambda^{3.4-7.5} & 0 & \lambda^{1.8-1.9} \end{pmatrix}$ |
| 50 | $\lvert h_u \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.3-6.2} & \lambda^{3.3-6.7} \\ \lambda^{2.3-6.2} & \lambda^{0.5-6.8} & \lambda^{0.9-7.6} \\ \lambda^{3.3-6.7} & \lambda^{0.9-7.6} & \lambda^{0.5-14.9} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.7-7.7} & \lambda^{2.4-5.0} \\ \lambda^{3.7-7.7} & \lambda^{2.7-5.5} & \lambda^{1.6-2.8} \\ \lambda^{2.4-5.0} & \lambda^{1.6-2.8} & \lambda^{0.5} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.3-8.5} & \lambda^{2.3-5.0} \\ \lambda^{4.3-8.5} & \lambda^{3.9-6.8} & \lambda^{2.6-3.4} \\ \lambda^{2.3-5.0} & \lambda^{2.6-3.4} & \lambda^{0.5} \end{pmatrix}$ |
| | $\lvert h_d \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.1-4.4} & 0 \\ \lambda^{2.1-4.4} & \lambda^{0.5-7.5} & \lambda^{1.0-3.4} \\ 0 & \lambda^{1.0-3.4} & \lambda^{0.5-5.2} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.2-5.4} & \lambda^{2.1-7.3} \\ \lambda^{3.2-5.4} & 0 & \lambda^{1.8-2.6} \\ \lambda^{2.1-7.3} & \lambda^{1.8-2.6} & \lambda^{0.5-0.6} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.6-5.6} & \lambda^{2.1-7.4} \\ \lambda^{3.6-5.6} & \lambda^{3.1-4.7} & 0 \\ \lambda^{2.1-7.4} & 0 & \lambda^{0.5-0.6} \end{pmatrix}$ |

## Minimal SUSY Extension of the Standard Model

$$\Lambda = 2 \times 10^{16}\ \mathrm{GeV}$$

| $\tan\beta$ | | $Y_{d13}=0$ | $Y_{d22}=0$ | $Y_{d23}=0$ |
|---|---|---|---|---|
| 10 | $\lvert Y_u \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.4-6.0} & \lambda^{3.2-6.4} \\ \lambda^{2.4-6.0} & \lambda^{0.4-4.5} & \lambda^{0.8-5.9} \\ \lambda^{3.2-6.4} & \lambda^{0.8-5.9} & \lambda^{0.4-12.4} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.7-9.1} & \lambda^{2.4-4.8} \\ \lambda^{3.7-9.1} & \lambda^{2.6-4.7} & \lambda^{1.5-2.4} \\ \lambda^{2.4-4.8} & \lambda^{1.5-2.4} & \lambda^{0.4-0.5} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.3-8.5} & \lambda^{2.4-4.9} \\ \lambda^{4.3-8.5} & \lambda^{3.9-6.0} & \lambda^{2.5-3.0} \\ \lambda^{2.4-4.9} & \lambda^{2.5-3.0} & \lambda^{0.4} \end{pmatrix}$ |
| | $\lvert Y_d \rvert$ | $\begin{pmatrix} 0 & \lambda^{3.5-5.7} & 0 \\ \lambda^{3.5-5.7} & \lambda^{1.9-7.4} & \lambda^{2.3-4.1} \\ 0 & \lambda^{2.3-4.1} & \lambda^{1.9-6.3} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.6-6.7} & \lambda^{3.5-7.7} \\ \lambda^{4.6-6.7} & 0 & \lambda^{3.1-3.6} \\ \lambda^{3.5-7.7} & \lambda^{3.1-3.6} & \lambda^{1.9} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{5.0-6.4} & \lambda^{3.5-7.2} \\ \lambda^{5.0-6.4} & \lambda^{4.4-5.4} & 0 \\ \lambda^{3.5-7.2} & 0 & \lambda^{1.9} \end{pmatrix}$ |
| 50 | $\lvert Y_u \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.3-5.8} & \lambda^{3.2-6.5} \\ \lambda^{2.3-5.8} & \lambda^{0.3-3.7} & \lambda^{0.8-7.2} \\ \lambda^{3.2-6.5} & \lambda^{0.8-7.2} & \lambda^{0.3-14.5} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.7-8.3} & \lambda^{2.3-4.7} \\ \lambda^{3.7-8.3} & \lambda^{2.6-5.1} & \lambda^{1.5-2.6} \\ \lambda^{2.3-4.7} & \lambda^{1.5-2.6} & \lambda^{0.3-0.4} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{4.3-8.2} & \lambda^{2.3-4.8} \\ \lambda^{4.3-8.2} & \lambda^{3.9-6.4} & \lambda^{2.5-3.2} \\ \lambda^{2.3-4.8} & \lambda^{2.5-3.2} & \lambda^{0.3-0.4} \end{pmatrix}$ |
| | $\lvert Y_d \rvert$ | $\begin{pmatrix} 0 & \lambda^{2.1-4.5} & 0 \\ \lambda^{2.1-4.5} & \lambda^{0.5-5.4} & \lambda^{0.9-2.8} \\ 0 & \lambda^{0.9-2.8} & \lambda^{0.5-5.2} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.3-5.5} & \lambda^{2.1-6.3} \\ \lambda^{3.3-5.5} & 0 & \lambda^{1.8-2.5} \\ \lambda^{2.1-6.3} & \lambda^{1.8-2.5} & \lambda^{0.5-0.6} \end{pmatrix}$ | $\begin{pmatrix} 0 & \lambda^{3.7-5.2} & \lambda^{2.1-6.2} \\ \lambda^{3.7-5.2} & \lambda^{3.1-4.6} & 0 \\ \lambda^{2.1-6.2} & 0 & \lambda^{0.5-0.6} \end{pmatrix}$ |

# Power Structures

## Imposing Small Rotations on the Yukawa Matrices

- **SM**

$$|h_u| \propto \begin{pmatrix} 0 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^{\frac{5}{2}} & \lambda \\ \lambda^4 & \lambda & 1 \end{pmatrix} \qquad |h_d| \propto \begin{pmatrix} 0 & \lambda^{\frac{7}{2}} & 0 \\ \lambda^{\frac{7}{2}} & \lambda^{\frac{5}{2}} & \lambda \\ 0 & \lambda & 1 \end{pmatrix}$$

- **MSSM**
  - $\tan\beta = 10$

$$|Y_u| \propto \begin{pmatrix} 0 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^3 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \qquad |Y_d| \propto \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$$

  - $\tan\beta = 50$

$$|Y_u| \propto \begin{pmatrix} 0 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^3 & \lambda \\ \lambda^4 & \lambda & 1 \end{pmatrix} \qquad |Y_d| \propto \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$$

# Conclusions

- We considered zeroes of the fundamental fermions
- Weak Basis Zeroes - no physical content
- Four-Zero Textures do have physical implications
- Four-Zero Textures seem not favorable at low energy
- Power Structures and Small Rotations