Flavor Symmetry Breaking and Vacuum Alignment on Orbifolds

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Based on Tatsuo Kobayashi and Koichi Yoshioka,
0809.3064[hep-ph]

outline

- The masses and mixing angles of quarks and leptons have been long-standing and inspiring problems.
- Flavor symmetry has been widely investigated.

  For example, S3, A4, D4, U(1) and SU(3) etc.
- We discuss flavor symmetric models which generate the masses and mixing angles of leptons.
- I will present a framework for handling flavor symmetry breaking.

Contents

1. Introduction

- In lepton sectors, large mass hierarchies and large mixing angles are observed. The following data are based on the recent experiments, such as KamLAND, CHOOZ, K2K and MINOS .

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056

(T.Schwetz, et al, arXiv 0808.2016)

- These results suggest that the large lepton mixing matrix is close to tri-bimaximal mixing, which is described as follows in the below base,

$$|\nu_\alpha\rangle = U_{\alpha i}\,|\nu_i\rangle\,, \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3,$$

$$U_{\mathrm{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

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$$U_{\mathrm{MNS}} = U_{\mathrm{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

- The physical meaning of the mixing matrix, named MNS matric, is the difference between the mass base of the charged leptons and of the neutrinos. In the base of diagonal charged lepton mass matrix, tri-bimaximal mixing gives the following neutrino mass matrix:

$$M_\nu = U_{tri} \begin{pmatrix} \hat{m}_1 & 0 & 0 \\ 0 & \hat{m}_2 & 0 \\ 0 & 0 & \hat{m}_3 \end{pmatrix} U_{tri}^T$$

It can be decomposed as follows.

$$= m_1(\hat{m}_i) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_2(\hat{m}_i) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + m_3(\hat{m}_i) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \tilde{m}_1(\hat{m}_i) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \tilde{m}_2(\hat{m}_i) \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \tilde{m}_3(\hat{m}_i) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Is this a clue of new physics?

Flavor symmetry exists behind this structure ?

If flavor symmetries are hidden, what is needed ?

- (Charged lepton sector) + (Neutrino sector) are

$$y_I^{c.lep} L^I H_d E^I + y_{IJ}^{\nu}\left(\langle\phi\rangle\right) L^I L^J H_u H_u$$

When charged lepton mass matrix is diagonal, ⬇

$$m_1(\langle\phi\rangle)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_2(\langle\phi\rangle)\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + m_3(\langle\phi\rangle)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

the symmetry and the VEVs of fields decide these forms.

- In order to realize the realistic mass matrices of lepton,
"appropriate" flavor symmetry should be chosen,
and the expectation values must take specific directions.
(Vacuum alignment)

A4 model ⟺

$$\begin{pmatrix} \langle \phi^1 \rangle \\ \langle \phi^2 \rangle \\ \langle \phi^3 \rangle \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} \langle \phi_s{}^1 \rangle \\ \langle \phi_s{}^2 \rangle \\ \langle \phi_s{}^3 \rangle \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- The vacuum alignment needs elaboration of analyzing **complicated** scalar potential. Higher-order couplings would change the minimum of potential. That causes the vacuum alignment moving.

Main purpose of this talk

- We present a framework for handling flavor symmetry breaking.
- This framework could make it **easier** and **systematic** to built realistic models with flavor symmetries.

2. Vacuum Alignment on Orbifolds

- We consider models with extra dimensions and flavor symmetry. In our models, there are bulk scalar fields in higher-dimensional space.
- The vacuum alignment is achieved by the boundary conditions of the scalar fields,

named Scherk-Schwarz twisted boundary conditions.

- This means that the physics is described by massless zero modes in the low-energy theory.

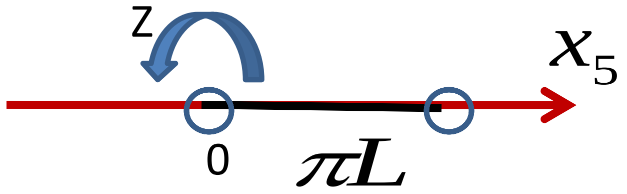
The directions of massless zero modes are fixed by the boundary conditions. The zero modes break flavor symmetries.

2-1. General recipe

- We discuss boundary conditions and zero modes.
- In a 5-dimensional S1/Z2 orbifold theory, there are 2 operations:

  The reflection : $\hat{Z} : x_5 \to -x_5$

  The translation : $\hat{T} : x_5 \to x_5 + 2\pi L$



- Orbifold S1/Z2 requires

  $\hat{Z}x_5 = x_5, \hat{T}x_5 = x_5$ which satisfy $\hat{Z}\hat{T} = \hat{T}^{-1}\hat{Z}$.

- These operations in the field space are

$$\hat{Z}\phi(x_5) = Z^{-1}\phi(-x_5), \hat{T}\phi(x_5) = T^{-1}\phi(x_5 + 2\pi L).$$

- The boundary conditions are defined by

$$\hat{Z}\phi(x_5) = \phi(x_5), \quad \hat{T}\phi(x_5) = \phi(x_5).$$

- Z and T must satisfy $ZT = T^{-1}Z$.

- Bulk scalar fields, $\phi(x_5)$, transform as triplets under flavor symmetries.

The boundary conditions for the scalars are

$$\hat{Z}: \quad \hat{Z}\phi(x_5) = Z^{-1}\phi(-x_5) = \phi(x_5),$$
$$\hat{T}: \quad \hat{T}\phi(x_5) = T^{-1}\phi(x_5 + 2\pi L) = \phi(x_5)$$

- Z and T are the <u>unitary representation matrices of flavor symmetry group</u>.
- The matrices must satisfy $Z^2 = (ZT)^2 = 1$.

- From the boundary conditions,

  the mode expansion is

$$\phi(x_\mu, x_5) = \sum_n e^{i\omega_n x_5} \phi^{(n)}(x_\mu)$$

$$\phi_i(x^\mu, x^5) = \sum_n \left(\exp\left[i\left(\frac{W}{2} + n\right)\frac{x^5}{L}\right]\right)_{ij} \phi_j^{(n)}(x^\mu).$$

where $Z = diag(+1, \pm 1, \pm 1)$ and $T = e^{\pi i W}$.

W is the hermite 3 × 3 matrix.

- For example, when Z and W take the form,

$$Z = \begin{pmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$$

we find a single massless mode$(\omega_0 = 0)$, $\langle \phi^{(0)} \rangle^T \propto (1 \quad 0 \quad 0)$.

- In the low-energy theory, the physics is described by massless modes. We find a single massless zero mode,

$$\left\langle \phi^{(0)} \right\rangle^T \propto \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}.$$

- This result suggests the direction of flavor symmetry breaking is fixed by the boundary condition of the bulk scalar field.
- The vacuum alignment is realized by finding the eigenvector with eigenvalue +1 of unitary representation matrices, such as Z and T.

3. A4 Model

<u>We consider A4 flavor symmetry as a example.</u>

- The fundamental elements of A4 are P and R,

which satisfy $P^2 = (PR)^3 = R^3 = 1.$

- A4 has three non-trivial representations,

the triplet and two pseudo singlets (1’ and 1”).

- For example, P and R for the triplet are described as

$$P = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}, \qquad R = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}.$$

3-1. Vacuum alignment

We consider a bulk scalar in the triplet of A_4 which lives in T2/(Z2 × Z3) orbifold.

- The extra-dimensional space has 4 operations,

$$\hat{T} \ : \ z \ \rightarrow \ z + 2\pi L, \qquad \hat{Z}_2 \ : \ z \ \rightarrow \ -z,$$

$$\hat{T}' \ : \ z \ \rightarrow \ z + 2\pi L\chi, \qquad \hat{Z}_3 \ : \ z \ \rightarrow \ \chi z,$$

where $z = x^5 + ix^6$ and $\chi = e^{2\pi i/3}$.

- On the field variables, the compactification is performed with the identification;

$$\phi(z + 2\pi L) = T\,\phi(z), \qquad \phi(-z) = Z_2\,\phi(z),$$

$$\phi(z + 2\pi L\chi) = T'\phi(z), \qquad \phi(\chi z) = Z_3\,\phi(z),$$

where T, T' and $Z_{2,3}$ are the triplet rep. matrices of A4.

- T, T' and $Z_{2,3}$ must satisfy relations for the T_2/Z_6, such as $(Z_2)^2 = (Z_3)^3 = I$.
- The zero mode is included in the eigenvectors with +1 eigenvalue of T ,T', Z2 and Z3.

$$\phi(z + 2\pi L) = T\,\phi(z), \qquad \phi(-z) = Z_2\,\phi(z),$$

$$\phi(z + 2\pi L\chi) = T'\phi(z), \qquad \phi(\chi z) = Z_3\,\phi(z),$$

Finally, we find 2 non-trivial boundary conditions,

(i) $Z_2 = P, \qquad Z_3 = T = T' = I$

The zere mode is $\langle\phi\rangle \propto (1,\,0,\,0).$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(ii) $Z_3 = R, \qquad Z_2 = T = T' = I$

The zere mode is $\langle\phi\rangle \propto (1,\,1,\,1).$

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \chi & 0 \\ 0 & 0 & \chi^2 \end{pmatrix}, \chi^3 = 1$$

3-2. Illustrative model (A_4 flavor symmetry)

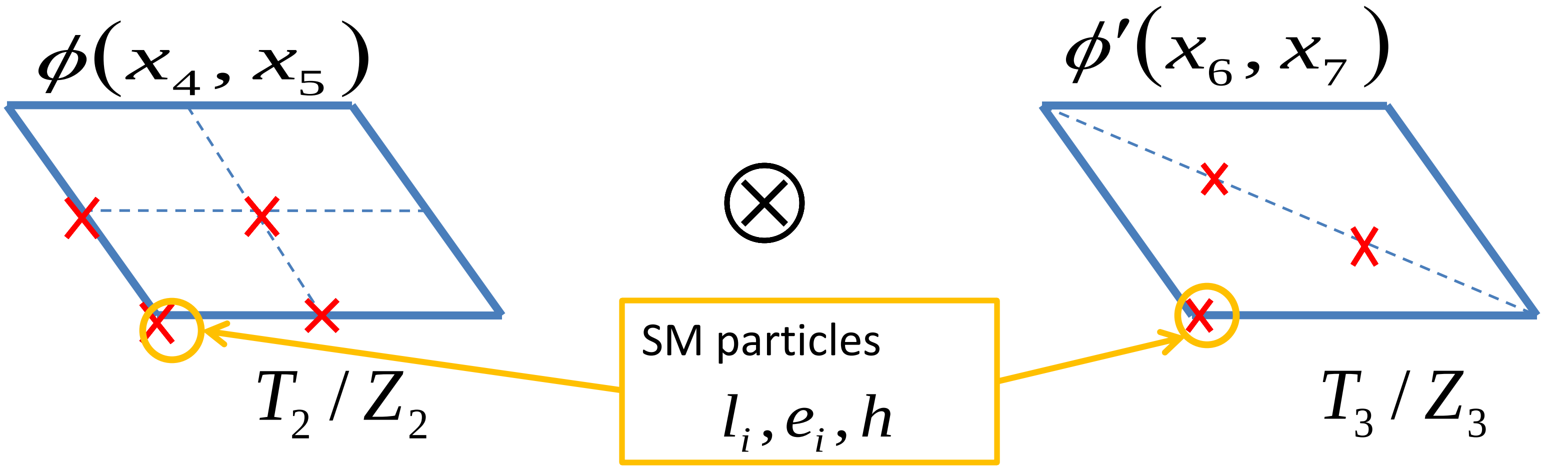
Based on the vacuum alignment, we construct an explicit orbifold model for lepton.

- We consider an 8-d theory on $T_2/Z_2 \times T_2/Z_3$.

➢ The standard-model fields live on a fixed point of the orbifold, such as the origin (z=0).

➢ There are 2 bulk scalars, ϕ and ϕ' for the triplet.

ϕ lives in T_2/Z_2, ϕ' lives in T_2/Z_3.



- The A4 and Z3 symmetry assignment is

	ℓ	e_1	e_2	e_3	h	η	ϕ	ϕ'
A_4	3	1	$1'$	$1''$	1	1	3	3
Z	χ	χ	χ	χ	1	χ	χ	1

Altarelli and Feruglio, Nucl.Phys. B741, 215(2006))

- The interactions under the flavor symmetry,

$$\mathcal{L}_Y = y_1 \bar{e}_1 \ell \phi' h + y_2 \bar{e}_2 \phi' \ell h + y_3 \bar{e}_3 \ell \phi' h + w_1 \phi \overline{\ell^c} \ell h^2 + w_2 \eta \overline{\ell^c} \ell h^2 + \cdots$$

The bulk fields have boundary conditions:

$$\begin{aligned} \phi(-z) &= P\,\phi(z), \\ \phi'(\chi z') &= R\,\phi'(z'), \end{aligned}$$

where $z = x^5 + ix^6$ and $z' = x^7 + ix^8$.

The boundary conditions leads the vacuum alignment,

$$\langle \phi \rangle = a\,(1,\,0,\,0), \qquad \langle \phi' \rangle = a'(1,\,1,\,1)$$

Charged lepton mass matrix ($v = \langle h \rangle$)

$$M_l = a' v \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix}$$

Each charged lepton mass is proportional to yi, so we need to use the other mechanism, such as Froggatt-Nielsen mechanism, in order to realize the hierarchy.

Neutrino mass matrix

correspond to tri-bimaximal.

$$M_\nu = \langle \eta \rangle v^2 \omega_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + a \left(\frac{\omega_1}{3} \right) v^2 \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

This form is fitted to the present experimental values. The eigenvalues are

$$\Delta m_{21}^2 = -\mathrm{Re}[w_1 a (w_1 a + 2 w_2 b)] v^2 / 2$$

$$\Delta m_{31}^2 = -4 \mathrm{Re}(w_1 w_2 a b) v^2$$

How about extension to supersymmetric model?

- The contribution to FCNC process should be suppressed.
- There is a possibility that vacuum minimumization leads the extra contribution.
- For example, in A4 model, the vacuum minimum is modified by the higher operators:

$$\phi' = a'(1,1,1) \Rightarrow \phi' = a'(1,1,1) + a'(\delta_1, \delta_2, \delta_3) \qquad \delta_i \cong a' / \Lambda$$

$$\Rightarrow \quad (m^2{}_{LR})_{12} \cong \delta_i m_\mu (A_1 - A_2)$$

$$\delta_i \cong m_\tau \tan\beta / m_t$$

H. Ishimori, T. Kobayashi, Y.O. , M. Tanimoto.
e-Print: **arXiv:0807.4625** [hep-ph]

This orbifold scenario does not induce this contribution without the assumption, $A_1 \cong A_2$.

4. Summary

- We have presented the scenario for breaking flavor symmetry with Scherk-Schwarz twisted boundary conditions on bulk scalar fields.
- The VEVs are aligned to definite directions.
- Our scheme needs no elaboration of analyzing complicated scalar potential.
- The higher-order terms do not change the directions, so that we can expect that their extra contributions to fermion mass matrices are suppressed and controlled by flavor symmetries.
- **The only thing we have to do is to find +1 eigenvalue and the eigenvector.** This may make it possible to discuss flavor symmetric models, which realize tri-bimaximal mixing, easily and systematically.

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