

Minicharges and Magnetic Monopoles

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(with Jörg Jäckel and Valya Khoze)

Outline

- 1 Kinetic mixing and minicharges
- 2 Minicharges and monopoles
- 3 Magnetic mixing
- 4 Conclusions

The most general* $U(1) \times U(1)'$ Lagrangian

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu})$$

*4d, renormalizable, gauge sector only, etc.

The most general* $U(1) \times U(1)'$ Lagrangian

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + 2\chi F_{\mu\nu}G^{\mu\nu})$$

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$$- \frac{1}{32\pi^2} (\theta_F F_{\mu\nu}\tilde{F}^{\mu\nu} + \theta_G G_{\mu\nu}\tilde{G}^{\mu\nu} + 2\theta_{FG} F_{\mu\nu}\tilde{G}^{\mu\nu})$$

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- Nonzero χ , kinetic mixing:
Important for charged particles (Minicharges)
- Nonzero θ s:
Important for **magnetic monopoles** (Witten effect, **Magnetic Mixing**)

In this talk:

- Can we have both minicharges and magnetic monopoles?
What about Dirac quantization?
- Can there be observable consequences of nonzero θ s?
→ Magnetic Mixing!

*4d, renormalizable, gauge sector only, etc.

Reminder: Minicharges

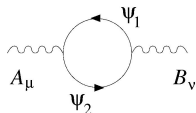
(→ talks by Redondo, Goodsell. . .)

$\theta\mathbf{s} = 0$ for now.

$U(1) \times U(1)'$ gauge theory, field strengths $F = dA$ and $G = dB$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{\chi}{2}F^{\mu\nu}G_{\mu\nu} - ej_{\mu}A^{\mu} - e'j'_{\mu}B^{\mu}$$

- **Kinetic mixing term** allowed by symmetries
- Even if $\chi = 0$ at tree level, $\chi \neq 0$ can be radiatively generated (if there are massive fields ψ_i charged under both $U(1)$ factors)



(→ Holdom '86)

- χ is arbitrary, irrational in general

Reminder: Minicharges

Diagonalize U(1) gauge fields:

$$C \equiv B + \chi A, \quad H \equiv dC = G + \chi F$$

$$\Rightarrow \mathcal{L} = -\frac{1}{4}(1 - \chi^2)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} - ej_{\mu}A^{\mu} - e'j'_{\mu}C^{\mu} + \chi e' j'_{\mu}A^{\mu}$$

Now current j' couples to gauge field A with charge $-\chi e'$.

Usual picture:

- A, F = photon and field strength of **electromagnetism**
- B, G = gauge field and field strength of **hidden sector**
- Hidden matter fields pick up EM charges $\sim \chi$: **minicharged particles**
- χ must be tiny to evade experimental constraints

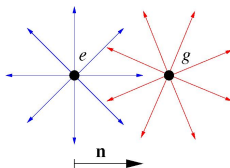
Another reminder: The Dirac Monopole

Dirac quantization condition for U(1) magnetic monopoles:

$$\frac{eg}{2\pi} \in \mathbb{Z}$$

g = magnetic, e = electric charge

One derivation (out of many): Static **electron-monopole** configuration



carries angular momentum:

$$\mathbf{L} = \int d^3x \mathbf{x} \times (\mathbf{E} \times \mathbf{B}) = \frac{eg}{4\pi} \mathbf{n}, \quad |\mathbf{L}| = \frac{eg}{4\pi}$$

Angular momentum quantization \Rightarrow Dirac quantization condition

Monopoles and electric charge quantization

If $eg/2\pi \in \mathbb{Z}$ for all electric charges e , then $e_1/e_2 \in \mathbb{Q}$ for any two charges $e_{1,2}$.

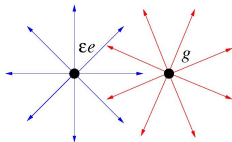
A puzzle: How is this compatible with minicharged particles?

Kinetic mixing generated radiatively \Rightarrow minicharges **arbitrary, irrational**

Example: For

- electron with electric charge e
- magnetic monopole with magnetic charge $g = 2\pi n/e$
- minicharged particle with electric charge $-\chi e' \equiv \epsilon e$

the static configuration



has angular momentum $|\mathbf{L}| = n\epsilon/2 \notin \frac{1}{2}\mathbb{Z}$!

This type of monopole cannot be consistently added to the theory.

A generalized Dirac quantization condition

Solution: Permit only monopoles with suitable magnetic charges **also under the hidden U(1)**.

Particle	q_{vis}	q_{hid}	g_{vis}	g_{hid}
electron	e	0	0	0
minicharged particle	$-\chi e'$	e'	0	0
monopole	0	0	g	g'

We have seen that $g' = 0$ is not allowed.

But: With

- $(g, g') = \left(0, \frac{2\pi n}{e'}\right)$
 - or $(g, g') = \left(\frac{2\pi m}{e}, \frac{2\pi \chi m}{e}\right)$
- $(m, n \in \mathbb{Z})$

the **total** field angular momentum

$$\mathbf{L} = \int d^3x \mathbf{x} \times (\mathbf{E}_{\text{vis}} \times \mathbf{B}_{\text{vis}} + \mathbf{E}_{\text{hid}} \times \mathbf{B}_{\text{hid}}) = \frac{eg - \chi e'g}{4\pi} + \frac{e'g'}{4\pi}$$

is half-integral for all configurations.

Minicharges and magnetic monopoles

In terms of original basis (F and G field strengths):

Field angular momentum gets contribution from kinetic mixing term,

$$\mathbf{L} = \int d^3x \mathbf{x} \times (\mathbf{E}_{\text{vis}} \times \mathbf{B}_{\text{vis}} + \chi (\mathbf{E}_{\text{hid}} \times \mathbf{B}_{\text{vis}} + \mathbf{E}_{\text{vis}} \times \mathbf{B}_{\text{hid}}) + \mathbf{E}_{\text{hid}} \times \mathbf{B}_{\text{hid}})$$

In that basis, allowed monopoles have F - and G -magnetic charges

- $(g, g') = (\frac{2\pi n}{e}, 0)$
- or $(g, g') = (0, \frac{2\pi m}{e'})$

corresponding to the F - and H -magnetic charges found before.

The 't Hooft-Polyakov monopole and kinetic mixing

Some models **necessarily contain** monopoles with **calculable** properties.

Example: $SU(2)$ gauge theory, broken to $U(1)$ by adjoint Higgs:

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} - \frac{1}{2}(D_\mu\phi)^a (D^\mu\phi)^a + m^2\phi^a\phi^a - \lambda(\phi^a\phi^a)^2$$

“Hedgehog vacuum”, $\langle\phi^a\rangle = r^a f(r)$: breaks $SU(2) \rightarrow U(1)$ at large r , magnetic monopole at $r = 0$ (\rightarrow 't Hooft, Polyakov '74)

Add an extra $U(1)$ with field strength F :

$$\Delta\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2M}\phi^a G^{\mu\nu a} F_{\mu\nu}$$

- Last term \Leftarrow integrated out heavy states charged under F and G
- Gives kinetic mixing between unbroken $U(1)' \subset SU(2)$ and extra $U(1)$:

$$\chi = \frac{|\langle\phi\rangle|}{M}$$

- **Can show: Monopole satisfies generalized Dirac quantization condition**

Yet another reminder: The Witten effect

(→ Witten '79)

A θ -term as in

$$\mathcal{L} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{32\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

gives **electric charges to magnetic monopoles**:

For magnetic monopole background (charge g) with superimposed static potential (A^0, \mathbf{A}),

$$-\frac{\theta}{32\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} = \frac{\theta}{8\pi^2} \mathbf{E} \cdot \mathbf{B} = \frac{\theta}{8\pi^2} (\nabla A^0) \cdot \left(\nabla \times \mathbf{A} + \frac{ge}{4\pi} \frac{\mathbf{r}}{r^3} \right)$$
$$L = \int d^3r \mathcal{L} \supset -\frac{\theta}{8\pi^2} \int d^3r A^0 \nabla \cdot \frac{ge}{4\pi} \frac{\mathbf{r}}{r^3} = -\frac{\theta eg}{8\pi^2} \int d^3r A^0 \delta^3(\mathbf{r})$$

RHS = electric point charge $-\theta eg/(8\pi^2)$, located at $\mathbf{r} = 0$, couples to electrostatic potential A^0

The monopole has **picked up an electric charge** $\sim \theta$.

Magnetic mixing

Our most general* $U(1) \times U(1)'$ Lagrangian had an off-diagonal θ -term:

$$\mathcal{L} \supset -\frac{\theta_{FG}}{16\pi^2} F_{\mu\nu} \tilde{G}^{\mu\nu}$$

A G -monopole will pick up an F -electric charge $\sim \theta_{FG}$
If F is electromagnetism, and G is a hidden sector field:

Hidden sector monopoles pick up ordinary electric minicharges.

Abelian θ -terms are usually ignored because visible sector monopoles are superheavy, $M \gg M_{\text{GUT}}$.

But hidden sector monopoles could be light if breaking scale is low or hidden sector is strongly coupled.

Radiatively generating magnetic mixing

Example with gauge group $U(1) \times SU(2)$, adjoint Higgs Φ

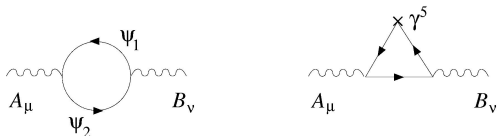
Add Dirac fermion Ψ with CP-violating masses and Yukawa couplings:

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} - (D\Phi)^2 - V(\Phi) + i\bar{\Psi}\not{D}\Psi - m_1\bar{\Psi}\Psi - im_2\bar{\Psi}\gamma^5\Psi - h_1\Phi\bar{\Psi}\Psi - ih_2\Phi\bar{\Psi}\gamma^5\Psi$$

After breaking to $U(1) \times U(1)'$: Ψ gets mass matrix

$$M = \begin{pmatrix} m_1 + im_2\gamma^5 & 0 \\ 0 & m_1 + im_2\gamma^5 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} h_1v + ih_2v\gamma^5 & 0 \\ 0 & -h_1v - ih_2\gamma^5 \end{pmatrix}$$

Kinetic and magnetic mixing induced by



- $m_2 \rightarrow$ diagonal θ -terms
- $h_1 \rightarrow$ kinetic mixing
- $h_2 \rightarrow$ magnetic mixing

Non-abelian embedding

$U(1)^k$ structure can come from single non-abelian gauge group

- Gauge group Higgsed to $U(1)^k$.
- Monopoles = 't Hooft-Polyakov monopoles
- Kinetic mixing generated by charged matter
- Magnetic mixing generated by charged matter with CP-violating couplings

Toy example: Seiberg–Witten theory with matter (\rightarrow Seiberg, Witten '94)

- $SU(N)$ gauge theory with $\mathcal{N} = 2$ SUSY
- adjoint Higgs from $\mathcal{N} = 2$ gauge multiplet breaks $SU(N) \rightarrow U(1)^N$ at generic point in moduli space
- gauge couplings and θ angles can be explicitly calculated (one-loop exact, non-perturbative corrections known)
- at strong coupling, **monopoles become light**

Tempting to speculate that this could happen in a realistic strongly coupled hidden sector — **could detect hidden monopoles by their minicharges!**

Conclusions

- Models with several $U(1)$ factors, or non-abelian gauge groups Higgsed to $U(1)^k$, naturally lead to minicharged particles
- They may still allow for magnetic monopoles if the Dirac quantization condition is suitably generalized
- If (one of) the $U(1)$ factors comes from a non-abelian gauge group, and the monopole is a 't Hooft-Polyakov monopole, then the modified Dirac quantization condition is automatically satisfied as expected
- Ordinary θ -terms \Rightarrow electric charges for magnetic monopoles: Witten effect
- Off-diagonal θ -terms \Rightarrow visible electric charges for hidden sector monopoles: magnetic mixing
- θ -terms are induced by CP-violating hidden matter fields
- A hidden sector monopole may be light (and thus detectable), e.g. if it is the 't Hooft-Polyakov monopole of a non-abelian gauge group
 - with a low breaking scale
 - or at strong coupling