Magnetized Orbifold Models

String Interactions and Flavor Symmetries

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Based on
0812.3534 Magnetized orbifold
0903.3800 Higher order coupling
0904.2631 Flavor symmetry
hep-th/0610026 Unification

With

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String compactification

String theory is a promising first principle giving rise to the Standard Model.
Procedures for obtaining a model. For a given string theory,

1. Compactify on a manifold with a suitable symmetry.
   ex. 10 - 6 = 4, $SU(3)$ or $\mathbf{Z}_N$ holonomy
2. Break the symmetry by associating the symmetries of the manifold with those of string theory.
   ex. noncontractable loop $\leftrightarrow$ gauge background
3. Vacuum configuration.
   ex. effective fields develop VEVs $\sim M_{\rm Pl}$.

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Low-energy interaction

is determined by the field distribution on internal geometry.

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Common feature:
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is determined by the field distribution on internal geometry.
A very interesting wavefunction:

$$\text{Jacobi theta function } \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (zM, \tau M)$$

already contains the the sufficient information of stringy nature?!

Compactification: common features

Property of 4D fermions, determined by Dirac operator

$$i(\gamma^\mu D_\mu + \gamma^m D_m)\Psi = 0$$

$$D_m = \partial_m + \underbrace{iA_m}_{\text{gauge}} + \frac{1}{2}\underbrace{\omega_m^{ab}}_{\text{geometry}}\gamma_{ab}$$

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Net # zero modes, e.g. in 6 extra dim

$$n_{\mathbf{R}} - n_{\overline{\mathbf{R}}} = \text{index}\, i\gamma^{m} D_m = \int_{\mathcal{M}^6} \underbrace{\text{Ch}(A)}_{\text{gauge}} \underbrace{\text{Todd}(\omega)}_{\text{geometry}}$$

$$= \int_{\mathcal{M}^6} (F \wedge F \wedge F + F \wedge R \wedge R)$$

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Caveats: extra dimensions R, magnetic fluxes F

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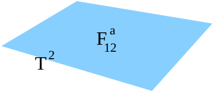
Chirality and charge are correlatied. $\mathbf{R}_L$ and $\overline{\mathbf{R}}_R$ CPT conjugates.
Some cases

- T^2 : $R = 0$ with $F \neq 0$
- $T^2/\mathbf{Z}_N$: $R \neq 0$ with $F \neq 0$

  ⋮
- $\mathbf{P}^1 \times \mathbf{P}^1, \mathbf{P}^2$ [Conlon Maharana Quevedo]
- Warped [Marchesano McGuirk Shiu] [Camara Marchesano]

Magnetized brane

Magnetic flux on T^2
:quantized 'monopole charges'



Background gauge field

$$F_{12} = \frac{2\pi}{\mathrm{Im}\tau} \begin{pmatrix} m_1 \mathbf{1}_{N_1} & & \\ & \ddots & \\ & & m_n \mathbf{1}_{N_n} \end{pmatrix}, \quad F_{34},\ F_{56}$$

describes

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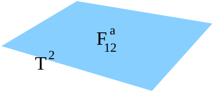

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describes

$$U(N) \to \prod U(N_i)$$

Fluction around the bg.

$$A = \begin{pmatrix} (\mathbf{N_i^2}, \mathbf{1}) & (\mathbf{N_i}, \mathbf{N_j}) \\ \times & (\mathbf{1}, \mathbf{N_j^2}) \end{pmatrix}$$

$$(\mathbf{N_i} + \mathbf{N_j})^2 \to (\mathbf{N_i^2}, \mathbf{1}) + (\mathbf{1}, \mathbf{N_j^2}) + (\mathbf{N_i}, \mathbf{N_j}) + CPT \text{ conj.}$$

Bifundamental.
Multiplicity: index$i\gamma^m D_m = M_{ij} \equiv m_i - m_j$.

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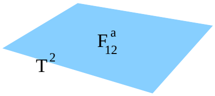

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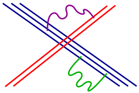
T-dual to intersecting branes



Wavefunctions

For given M, we have $|M|$ zero modes

$$\psi^{j,M}(z) = N_M G_M \, \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (Mz, \tau M)$$

$$j = 1, \ldots, |M|$$

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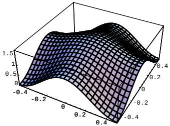

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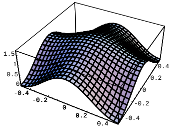

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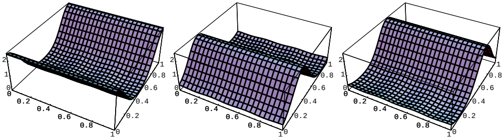

$M = 1 : \quad |\psi^{0,1}|^2 \quad |\psi^{-1,1}|^2 \quad |\psi^{1,1}|^2$

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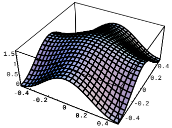

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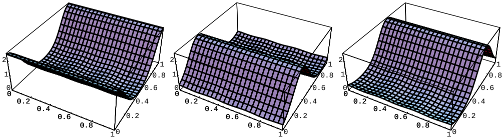

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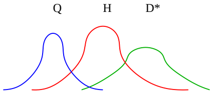
The normalization N_M automatically contains Kähler contribution [Cremades Ibanez Marchesano] [Di veccia Liccardo Marotta Pezzella]

$$N_M = \left(\frac{2\mathrm{Im}\tau |M|}{A^2} \right)^{1/4}, \quad G_M(z) = e^{i\pi M z \mathrm{Im}\ z / \mathrm{Im}\ \tau}$$

Yukawa coupling

Reuction from 10D SYM $\overline{\psi}[A, \psi]$.
Obtained from wavefunction overlap



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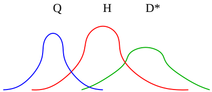

$$y_{ijk} = \int d^2z \, \psi^{i,M_1}(z)\psi^{j,M_2}(z)\psi^{k,M_3}(z) \text{ [Cremades Ibanez Marchesano]}$$

$$= \sum_{m \in \mathbf{Z}_{M_3}} \delta_{i+j+M_1 m,k} \, \vartheta \begin{bmatrix} \frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 M_3} \\ 0 \end{bmatrix} (0, \tau M_1 M_2 M_3)$$

Jacobi theta function $\vartheta[{}^a_b](\nu, \tau) = \sum_n \exp\left[\pi i (n+a)^2 \tau + 2\pi i (n+a)(\nu + b)\right]$.

- Size: how much the wavefns overlap
- Flavor symmetry: regularity of the wavefns

Reproduces the stringy nature

$$y_{ijk} = \int d^2z\, \psi^{i,M_1}(z)\psi^{j,M_2}(z)\psi^{k,M_3}(z)$$
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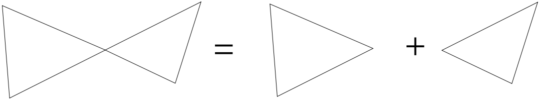
Reproduces CFT caclulation of string theory [Dixon Friedman Martinec Shenker] ...

$$Y_{ijk} = \langle V_1V_2V_3\rangle = \mathcal{Z}_{\mathrm{qu}}\cdot\sum_{\{X_{\mathrm{cl}}\}} e^{-S(X_{\mathrm{cl}})}$$

1. Stringy selection rule

$$i+j+k=0,\ \text{each modulo } M_1, M_2, M_3, \text{resp.}$$

2. Classical part reproduced: 'area law' $e^{-S(X_{\mathrm{cl}})} = e^{\mathrm{area}/4\pi\alpha'}$



3. Quantum part reproduced: normalization exactly match.

From the target space modular property $SL(2,\mathbf{Z})$.

Higher order coupling [Abe KSC Kobayashi Ohki]

$$y_{ijkl} = \int_{\text{internal}} d^2z \, \psi^{i,M_1}(z)\psi^{j,M_2}(z)\psi^{k,M_3}(z)\psi^{l,M_4}(z)$$

Important information on vacuum configuration. [Font Ibanez Quevedo Sierra] [Buchmuller Hamaguchi Lebedev Ratz] ...

CFT calculation exact, but in terms of less-known functions. [Atick Dixon Griffin Nemeschansky] [KSC Kobayashi]

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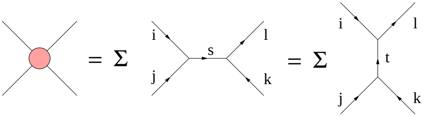
Inserting the completeness rel.

$$\psi^{s,M}(z)(\psi^{t,M}(z'))^* = \delta^{st}\delta^2(z-z')$$

expanded in terms of the 3-point couplings.



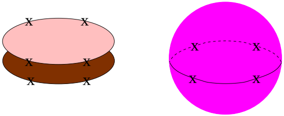
+ similar for u

Associatativity: channel independence

Extension to heterotc string [Kawai Lewellen Tye] [Abel Owen]

$$A_{\text{closed}} \sim A_{\text{open}}^2$$

Extension to intersecting branes: T-dual $A = \frac{1}{2\pi\alpha'}X$



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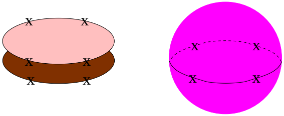
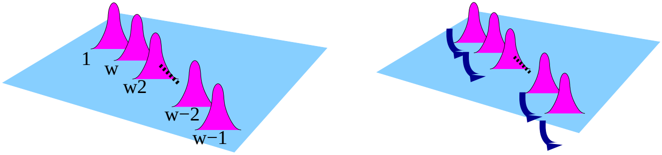


Any low-energy string interaction is calculated just by wavefn overlap
up to symmetric factor.

cf. DBI expansion unknown.

Flavor symmetry [Abe KSC Kobayashi Ohki]

Repetition of zero mode fermions with nontrivial configuration and charges
Selection rules $i + j + k = 0 \mod g = \gcd(M_1, M_2, M_3)$



$$\begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \ddots & \\ & & & \omega^{g-1} \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & & & \ddots \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega^g = 1$$

generate $(\mathbf{Z}_g \times \mathbf{Z}_g) \ltimes \mathbf{Z}_g$ of orger g^3, isomorphic to

1. $g = 2 : D_4$
2. $g = 3 : \Delta(27)$

New flavor symmetry

New symmetry $\Delta(27)$ found only in open string model

$$(\mathbf{Z}_g \times \mathbf{Z}_g) \ltimes \mathbf{Z}_g$$

Generators

$$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \omega^3 = 1$$

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Representations

M	Representation of $\Delta(27)$
3	$\mathbf{3}$
6	$2 \times \bar{\mathbf{3}}$
9	$\mathbf{1}_1,\ \mathbf{1}_2,\ \mathbf{1}_3,\ \mathbf{1}_4,\ \mathbf{1}_5,\ \mathbf{1}_6,\ \mathbf{1}_7,\ \mathbf{1}_8,\ \mathbf{1}_9$
12	$4 \times \mathbf{3}$
15	$5 \times \bar{\mathbf{3}}$
18	$2 \times \{\mathbf{1}_1, \mathbf{1}_2, \mathbf{1}_3, \mathbf{1}_4, \mathbf{1}_5, \mathbf{1}_6, \mathbf{1}_7, \mathbf{1}_8, \mathbf{1}_9\}$

$$M = 3 \qquad \mathbf{3} = \begin{pmatrix} \psi^{0,3} \\ \psi^{1,3} \\ \psi^{2,3} \end{pmatrix},$$

$$M = 6 \quad \bar{\mathbf{3}} = \begin{pmatrix} \psi^{0,6} \\ \psi^{2,6} \\ \psi^{4,6} \end{pmatrix} \begin{matrix} 1 \\ \omega^2 \\ \omega^4 \end{matrix}, \quad \bar{\mathbf{3}} = \begin{pmatrix} \psi^{3,6} \\ \psi^{5,6} \\ \psi^{1,6} \end{pmatrix} \begin{matrix} \omega^3 \\ \omega^5 \\ \omega^1 \end{matrix}.$$

With vanishing Wilson lines, the symmetry is enhanced to

$$(\mathbf{Z}_g \times \mathbf{Z}_g) \ltimes \mathbf{Z}_g \times \mathbf{Z}_2$$

1. $g = 2 : D_4 \longrightarrow D_4 \times \mathbf{Z}_2$
2. $g = 3 : \Delta(27) \longrightarrow \Delta(54)$

$$Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

$\Delta(54)$: Similar behavior from the closed (twisted) strings in $T^2/\mathbf{Z}_3$ orbifold. [Kobayashi, Raby, Zhang] [Kobayashi, Ploeger, Nilles, Raby, Ratz]

Magnetized orbifold

With the ingredients of extra dimimensions and magnetic fluxes, we can make varous effective theories.

Magnetized orbifold is useful bottom-up approach [Abe Kobayashi Ohki]

New moduli space

- Toroidal compactification:
  # zero modes $= M$ the difference of the magnetic flux
- Orbifold with associated projections:
  # zero modes $\neq M$ the difference of the magnetic flux

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ex. $T^2/\mathbf{Z}_2 : z \to -z$

$$\mathbf{Z}_2 : \Psi^j \leftrightarrow \Psi^{M-j}$$

$$\Psi_\pm = \frac{1}{\sqrt{2}}(\Psi^j \pm \Psi^{M-j})$$

The number of zero modes

M	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4

- Different Yukawa textures.
- Stringy completion not yet clear.

UV Completion

Global consistency ‘tadpole cancellation’ condition guarantees anomaly cancellation.

- Orientifold planes: charge sum the same as type I string.
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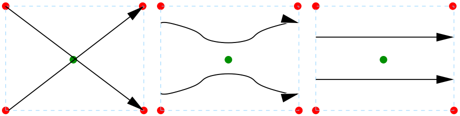
Recombination to type I string [KSC, J.E.Kim] [KSC]



- SUSY preserving marginal deformation
- energy costless
- local topology change [Hashimoto, Taylor] [Douglas, Zhou]
- growing size of D0-D4 instanton

Conclusions

Interaction among wavefunctions, obtained from string compactification. Characterized by

extra dimensions and magnetic fluxes

- Zero mode wavefun of Dirac eq.
  Jacobi theta function
  Reproduces all the stringy effect visible in the low-energy.
- Higher order coupling
  Important for vaccuum configuration
  expanded in terms of three-point couplings.
- Interaction determined by the geometry of extra dimension
  New flavor symmetries $D_4, \Delta(27), \ldots$
- Magnetized orbifold is a promising bottom-up approach.