Fixing D7 Brane Positions by F-Theory Fluxes

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A. Braun, A. Hebecker, CL, R. Valandro,
Nucl.Phys.B815:256-287,2009 [arXiv:0811.2416 [hep-th]]

Motivation

- F-Theory: Nonperturbative version of type IIB string theory [Vafa;Sen]
- Add two auxiliary dimensions, singularities of compactification manifold encode brane positions
- Recently, lots of interest in F-theory for (GUT) model building [Beasley,Heckman,Vafa; Donagi,Wijnholt; Marsano,Saulina,Schäfer-Nameki; Bourjaily; Tatar,Watari,Hayashi,Kawanao,Tsuchiya; . . .]
- Local models do not address global constraints like tadpole cancellation [Andreas,Curio]
- Four-form flux can stabilise moduli, including brane positions
- Simple example: F-Theory on $K3 \times \widetilde{K3}$, where $\widetilde{K3}$ is an elliptic fibration over $\mathbb{P}^1$ [Görlich et al.;Lust et al.; Aspinwall,Kallosh;Dasgupta et al.]
- Includes as special case the type IIB orientifold $K3 \times T^2/\mathbb{Z}_2$ [Angelantonj et al.]

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F-Theory/M-Theory Duality

M-Theory on $X_6 \times T^2$ → Compactify one S^1 → Type IIA on $X_6 \times S^1_A$ → T-dualise along S^1_A → Type IIB on $X_6 \times S^1_B$ $(R_B = 1/R_A)$

Fibrewise duality: $X_6 \times T^2 \rightsquigarrow$ elliptically fibred CY_4
dual to type IIB on base of fibration

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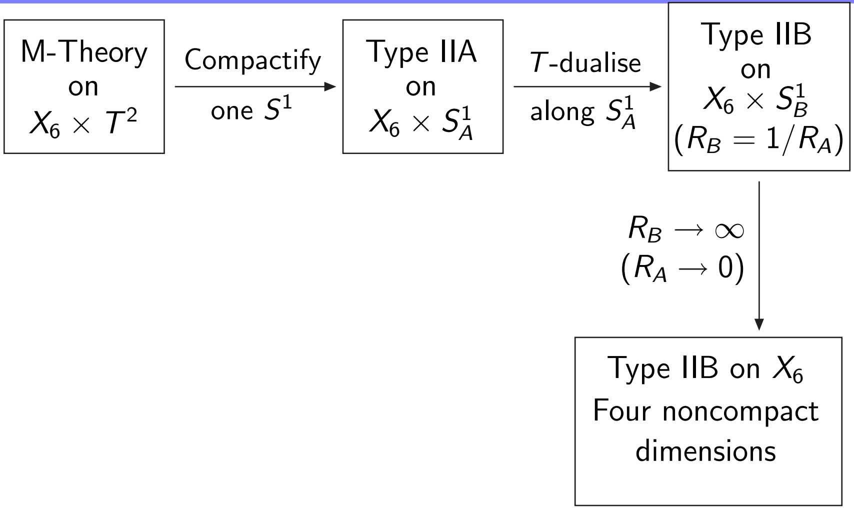

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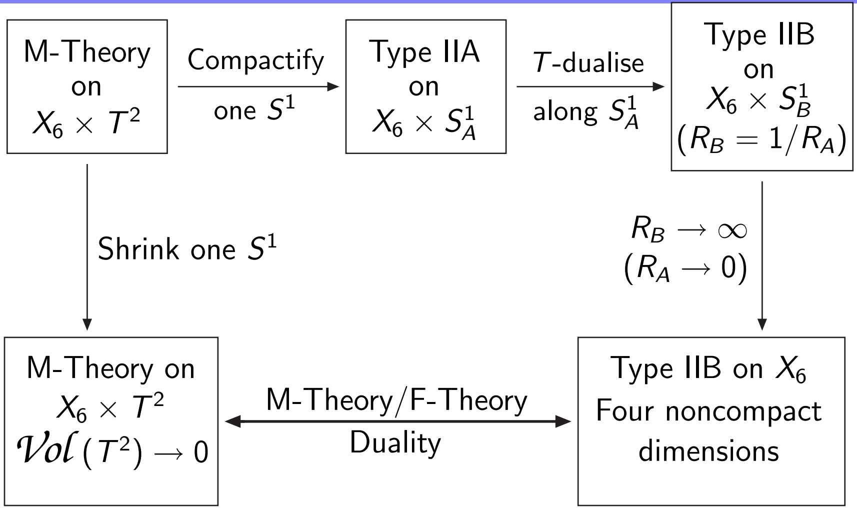

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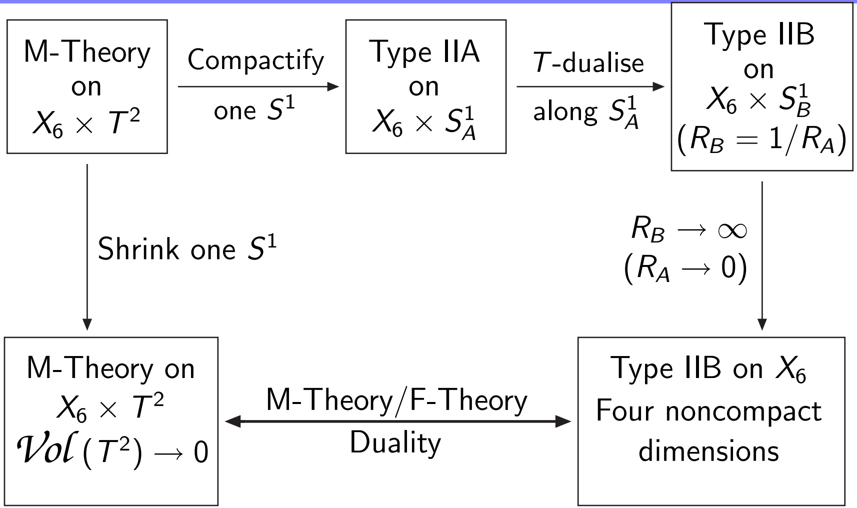

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K3: Calabi–Yau Two-Fold

- $H^2(K3, \mathbb{R})$ has signature $(3, 19)$
- Holomorphic two-form and Kähler form spanned by three real forms ω_i with $\omega_i \cdot \omega_j = \delta_{ij}$ and overall volume ν:

$$\omega = \omega_1 + \mathrm{i}\omega_2 \qquad j = \sqrt{2\nu}\,\omega_3$$

- K3 is hyperkähler: $SO(3)$ symmetry rotating the ω_i
$\rightsquigarrow$ geometry fixed by positive-norm three-plane $\Sigma \subset H^2(K3, \mathbb{R})$ and ν
- Moduli space: Rotation of the ω_i orthogonal to Σ and volume
$\rightsquigarrow$ dimension$= 3 \times 19 + 1 = 58$

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Cycles and Intersection Form

- There is an integral basis for $H^2(K3)$ such that intersection matrix is

$$U \oplus U \oplus U \oplus (-E_8) \oplus (-E_8) \ ,$$

where $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and E_8 is Cartan matrix of E_8

- The "U blocks" contain the positive norm directions

$\Rightarrow$ The ω_i must have components along the U blocks

Components along "E_8 directions" determine gauge group

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  - intersection matrix $\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$
  - $B \cdot \omega = F \cdot \omega = 0$

$\Rightarrow$ $(B + F, F)$ spans a U block, and we can parametrise the Kähler form as

$$j = b\,B + f\,F + c^a u_a \qquad \text{(where } u_a \cdot \omega = 0\text{)}$$

- F-theory limit: Fibre volume shrinks to zero $\Rightarrow b \to 0$. $K3$ volume is $\nu \sim bf - c^a c^a$, so we have to take $c^a \to 0$ as fast as $\sqrt{b}$ (as intuitively expected)
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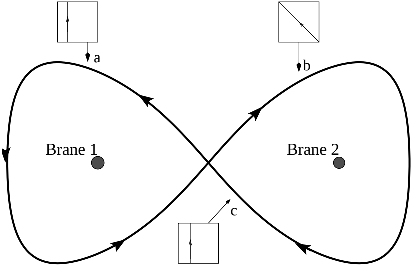
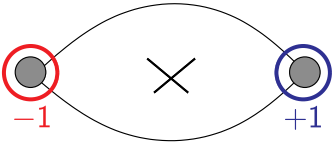
Cycles Between Branes

[Braun,Hebecker,Triendl]

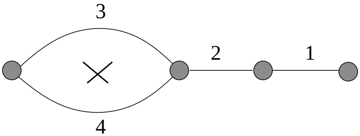




- One leg in the base, one in the fibre torus
- Shrink to zero when the branes are moved on top of each other.
- They are topologically a sphere $\leftrightarrow$ self-intersection -2.
- Cycles meeting at a brane intersect once, cycles encircling O planes ($\times$) do not intersect

Shrinking Cycles and Gauge Enhancement

Intersection matrix of shrinking cycles determines gauge group:
Consider e.g. $T^2/\mathbb{Z}_2$ orientifold: One O7, four D7s $\rightsquigarrow SO(8)$



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In appropriate basis, complex structure of $\widetilde{K3}$ is [Braun,Hebecker,Triendl]

$$\omega = \frac{\alpha}{2} + u\, e_2 + s\, \frac{\beta}{2} - \left(u\, s - \frac{z^2}{2} \right) e_1 + z_I \widehat{E}_I$$

Explicit mapping between complex structure and brane positions!

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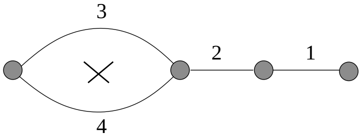

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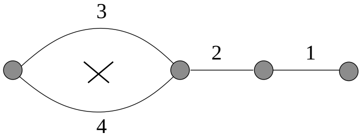

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base complex structure (u)

axiodilaton (s)

brane positions, $z_I = 0$ is $SO(8)^4$ point (z_I)

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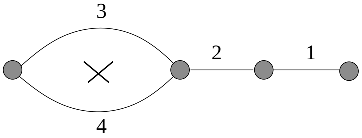

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Flux Potential

- Type IIB: Three-form flux G_3 on the bulk, two-form gauge flux F_2 on the branes can stabilise geometric and brane moduli
- In M-theory, these are combined into four-form flux G_4 (brane moduli become four-fold geometric moduli)
- Consistency conditions:
  - Flux quantisation: flux needs to be integral
  - Tadpole cancellation (without spacetime-filling M2 branes)

$$\frac{1}{2}\int_{K3\times\widetilde{K3}} G_4 \wedge G_4 = \frac{\chi}{24} = 24$$

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- Flux potential ($\mathcal{V}$ is the volume) : [Haack,Louis]

$$V = \frac{1}{4\mathcal{V}^3}\left(\int_{K3\times\widetilde{K3}} G_4 \wedge * G_4 - \frac{\chi}{12}\right)$$

- $K3 \times \widetilde{K3}$ is not a proper CY_4: Holonomy is $SU(2) \times SU(2)$
- G_4 induces map $G : H^2(\widetilde{K3}) \to H^2(K3)$ and its adjoint G^a by

$$G\tilde{\eta} = \int_{\widetilde{K3}} G_4 \wedge \tilde{\eta} \qquad G^a\,\eta = \int_{K3} G_4 \wedge \eta$$

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$K3 \times \widetilde{K3}$ Flux Potential

$$V = -\frac{1}{2(\nu \cdot \widetilde{\nu})^3} \left(\sum_j \| G\, \tilde{\omega}_j \|_\perp^2 + \sum_i \| G^a \omega_i \|_{\widetilde{\perp}}^2 \right)$$

$\|\cdot\|_\perp^2$ is the norm orthogonal to the three-plane

- Positive definite potential
- Manifestly symmetric under $SO(3)$
- Minima at $V = 0$:

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Minima: Existence, Flat Directions

- Minkowski minima do not necessarily exist: $G^a G$ must be diagonalisable and positive semi-definite (not guaranteed although $G^a G$ is self-adjoint, since metric is indefinite!)
- Flat directions generally exist and are desired: M-theory moduli become part of 4D vector fields in F-theory limit $\rightsquigarrow$ fixing these moduli breaks the gauge group (rank-reducing)
- Flux also induces explicit mass term for three-dimensional vectors
- Vacua can preserve $\mathcal{N} = 4$, $\mathcal{N} = 2$ or $\mathcal{N} = 0$ supersymmetry in four dimensions, depending on the action of G on the three-plane

Stabilisation Strategy

- F-theory limit fixes Kähler form (up to base volume), $j = f\,F$
- Holomorphic two-form determines shrinking cycles, i.e. gauge enhancement
- To stabilise a desired brane configuration:
  - Identify set of shrinking cycles to obtain desired brane stacks
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- Moving one brane off a stack. $\rightsquigarrow SO(8)^3 \times SO(6) \times U(1)$ or $SO(8)^3 \times SO(6)$
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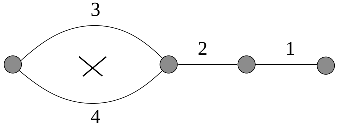

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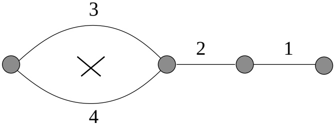

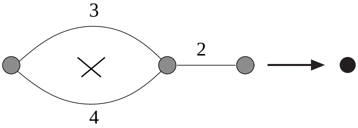

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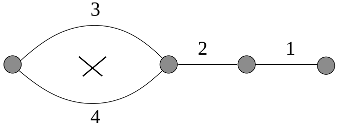

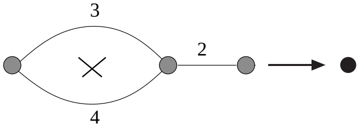

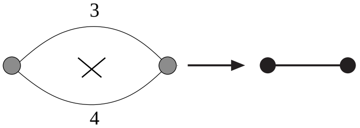

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