Causality of Holographic Hydrodynamics

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Motivation:

typically...

Use *String Theory* in a context of Maldacena correspondence as a guiding principle in constructing *Non-equilibrium Quantum Field Theory*

in this talk...

Use phenomenological hydrodynamics to constrain fundamental parameters of the QFT that allow for a dual holographic description

$\Rightarrow$ constraint possible central charges of a 4-dim CFT holographically dual to a Gauss-Bonnet gravity

Outline of the talk:

- Causality constraints on the transport coefficients of the hydrodynamics
- Holographic gauge/string duality *derives* hydrodynamics from first principles, thus mapping causality constraints onto the fundamental parameters of the theory.
- Results for CFT's holographically dual to Gauss-Bonnet gravity
  $\Rightarrow$ hydrodynamic approximation
  $\Rightarrow$ beyond hydrodynamic approximation
- Conclusions and future directions

Causality constraints on the transport coefficients of the hydrodynamics

Hydrodynamics is an effective theory describing near-equilibrium phenomena in (relativistic) QFT:

$$\nabla_\nu T^{\mu\nu} = 0$$

The stress-energy tensor includes both an equilibrium part (ϵ and P terms) and a dissipative part $\Pi^{\mu\nu}$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \Pi^{\mu\nu}\,.$$

where u^μ is a local 4-velocity of the fluid and

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu\,, \quad \Pi^\mu{}_\nu u^\nu = 0\,, \quad u^\mu \nu_\mu = 0\,,$$

Effective hydrodynamic description is equivalent to a derivative expansion of $\Pi^{\mu\nu}$ in local velocity gradients

Thus, to linear order in the derivative expansion

$$\Pi^{\mu\nu} = \Pi_1^{\mu\nu}(\eta, \zeta) = -\eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}(\nabla_\alpha u^\alpha)$$

($\sigma^{\mu\nu} \propto \nabla_\nu u^\mu$) with $\{\eta, \zeta\}$ being the viscosity coefficients.

To simplify further discussion we consider only CFT's from now on: $\zeta = 0\,, \epsilon = 3P$. To second order in the derivative expansion

$$\Pi^{\mu\nu} = \Pi_1^{\mu\nu}(\eta) + \Pi_2^{\mu\nu}(\eta, \tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3)$$

$$= -\eta\sigma^{\mu\nu} - \eta\tau_\Pi\left[\langle u\cdot\nabla\sigma^{\mu\nu}\rangle + \frac{1}{3}\left(\nabla\cdot u\right)\sigma^{\mu\nu}\right] + \mathrm{non-linear\,terms} + \cdots$$

$\Rightarrow$ It is straightforward to study dispersion relation of the linearized fluctuations in above theory

The dispersion relation of the shear channel fluctuations is given by

$$0 = -\mathfrak{w}^2 \, \tau_\Pi T - \frac{i\mathfrak{w}}{2\pi} + \boldsymbol{k}^2 \, \frac{\eta}{s} \, ,$$

where $\mathfrak{w} = \omega/(2\pi T)$ and $\boldsymbol{k} = k/(2\pi T)$. Now the speed with which a wave-front propagates out from a discontinuity in any initial data is governed by

$$\lim_{|\boldsymbol{k}|\to\infty} \left. \frac{\mathrm{Re}(\mathfrak{w})}{\boldsymbol{k}} \right|_{[\mathrm{shear}]} = \sqrt{\frac{\eta}{s \, \tau_\Pi T}} \equiv v^{front}_{[\mathrm{shear}]} \, .$$

Hence causality in this channel imposes the restriction

$$\tau_\Pi T \; \geq \frac{\eta}{s} \, .$$

Notice: the first-order hydrodynamics is recovered in the limit $\tau_\Pi \to 0$, so causality is always violated at this order in the derivative truncation

Similar considerations in the sound channel imposes the (more stringent) restriction

$$\tau_\Pi T \; \geq 2\frac{\eta}{s} \, .$$

CFT dual for a Gauss-Bonnet (GB) gravity

Gravitational action for a GB gravity with a negative cosmological constant

$$\mathcal{I} = \frac{1}{2\ell_P^3}\int d^5x\sqrt{-g}\left[\frac{12}{L^2} + R + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

is solvable for any λ_{GB} *i.e.,* , we can find exact (analytic) black hole solution and study its near-equilibrium properties.

Computing the boundary stress-energy tensor of $\mathcal{I}$

$$\langle T_\mu{}^\mu\rangle = \frac{c}{16\pi^2}I_4 - \frac{a}{16\pi^2}E_4$$

where the Euler density E_4 and the Weyl curvature I_4, respectively

$$E_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2\,, \qquad I_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

we identify a dual gauge theory as a CFT with central charges $\{c, a\}$ given by

$$c = \frac{\pi^2}{2^{3/2}} \frac{L^3}{\ell_P^3} (1+\sqrt{1-4\lambda_{GB}})^{3/2} \sqrt{1-4\lambda_{GB}}$$

$$a = \frac{\pi^2}{2^{3/2}} \frac{L^3}{\ell_P^3} (1+\sqrt{1-4\lambda_{GB}})^{3/2} \left(3\sqrt{1-4\lambda_{GB}}-2\right)$$

or

$$\frac{c-a}{c} = 2\left(\frac{1}{\sqrt{1-4\lambda_{GB}}}-1\right)$$

$\Rightarrow$ It is straightforward to study dispersion relation of the quasinormal modes of the GB black holes — these quasinormal modes are dual to linearized fluctuations in plasma (the shear and the sound channel modes)

For the shear viscosity one finds (Brigante *et.al*)

$$\frac{\eta}{s} = \frac{1}{4\pi}\Big[1-4\lambda_{GB}\Big]$$

For the relaxation time:

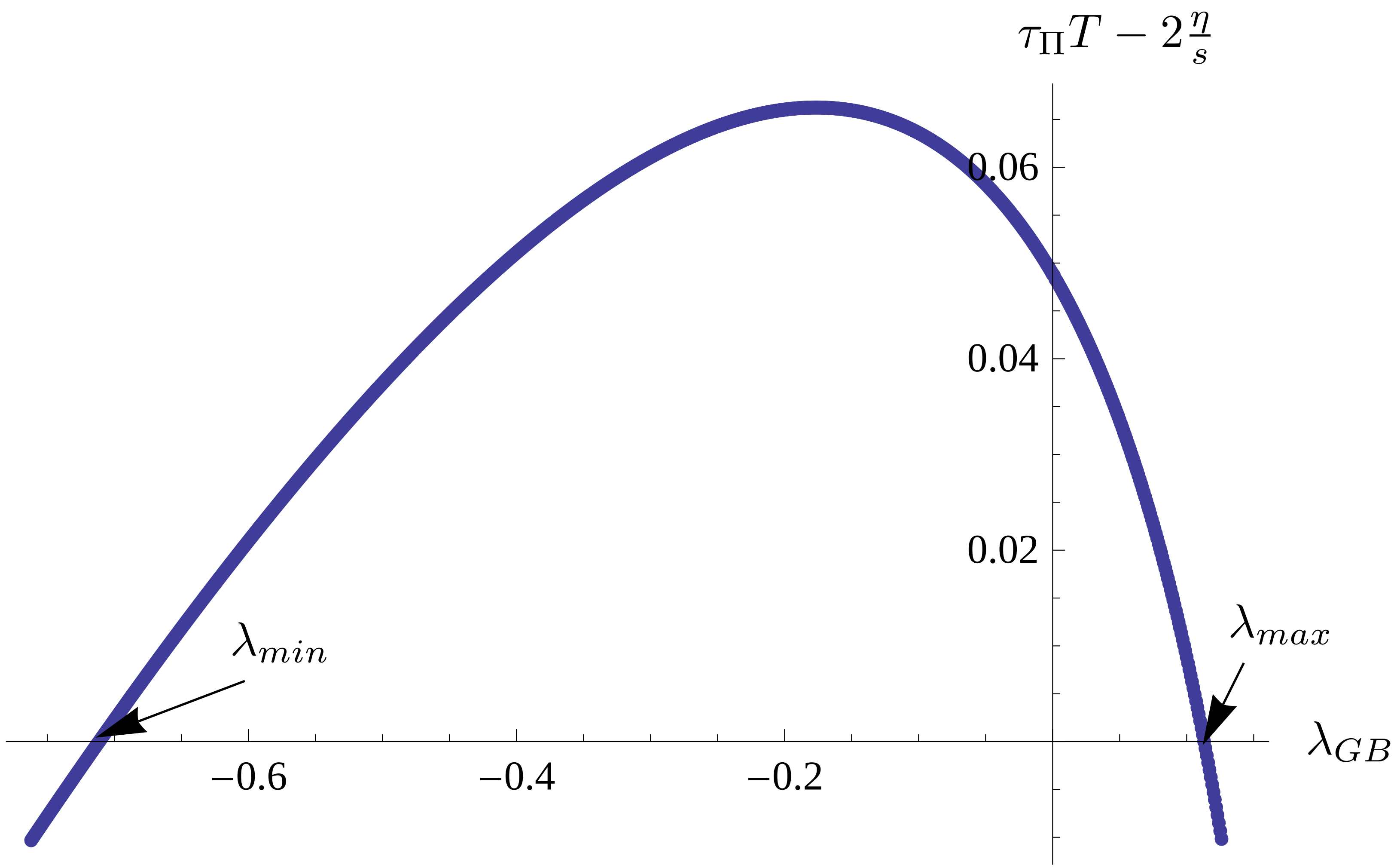

Figure 1: Causality of the second-order Gauss-Bonnet hydrodynamics is violated once $\tau_\Pi T < 2\frac{\eta}{s}$. Thus, $\lambda_{GB} \in [\lambda_{min}, \lambda_{max}]$, where $\lambda_{min} = -0.711(2)$ and $\lambda_{max} = 0.113(0)$.

Beyond hydrodynamic approximation in GB gravity

$\Rightarrow$ Previous analysis were based on the effective theory of the successive local-velocity derivative expansion in GB plasma

$\Rightarrow$ However, given the gravity dual we can study dispersion relation of quasinormal modes in GB plasma without any reference to a hydrodynamic expansion!

In this way we find that causality is violated in GB CFT plasma, unless

$$-\frac{7}{36} \le \lambda_{GB} \le \frac{9}{100}$$

which translates into the following constrain on the CFT central charges

$$-\frac{1}{2} < \frac{c-a}{c} < \frac{1}{2}$$

Conclusions

- We demonstrated that once hydrodynamics is derived from first principles, its causality would typically constrain fundamental parameters of the theory — there are many theories in holographic swampland!
- Rather satisfying, we find that the 'exact' constraint on the parameters of the CFT coming from causal propagation of linearized fluctuation are more stringent then the constraints from the hydrodynamic approximation

$$\text{second-order hydrodynamics:} \; -0.711 < \lambda_{GB} < 0.113$$

$$\text{exact hydrodynamics:} \; -\frac{7}{36} < \lambda_{GB} < \frac{9}{100}$$

Thus, it appears, that once the theory is fundamentally sound, *i.e.,* , embeddable in String Theory, its hydrodynamic description will be causal.

- Because λ_{GB} can be positive, the KSS viscosity bound can be violated (discussed in the literature previously)

Future directions:

There is much more to the CFT than just central charges!

Can AdS/CFT be used to constraint other parameters of the CFT?