Holography of Chern-Simons matter theories with flavour

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in colaboration with Stefan Hohenegger,
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- prerequisite for making the ABJM theory more interesting for possible phenomenological applications (e.g. in condensed matter physics) is the introduction of light matter fields in the fundamental representation
- in this talk: holography of ABJM CSM theory with flavour

Outline

1. Chern-Simons theory with flavour
   - $\mathcal{N} = 6$ ABJM Chern-Simons matter theory (review)
   - $\mathcal{N} = 3$ Chern-Simons Yang-Mills theory with flavour
2. Brane setup & lift to M-theory
   - Type IIB brane setup
   - T-dual type IIA setup & lift to M-theory
   - Hypertoric geometry with $SO(4)$ isometry
3. Probe D6-branes in $AdS_4 \times \mathbb{CP}^3$
   - Embedding of the D6-branes
   - Supersymmetry and stability
4. Conclusions & Outlook

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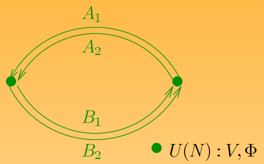
- gauge group $U(N)_k \times U(N)_{-k}$



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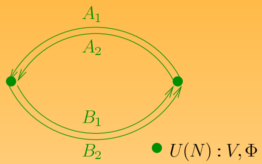

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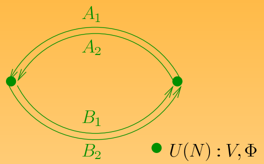

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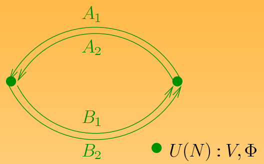

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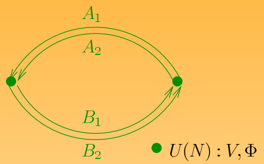

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- $\Phi_{1,2}$ massive, integrate out, get Klebanov-Witten-type superpotential

$$W^{\text{IR}}_{\text{ABJM}} = \frac{4\pi}{k}\,\mathrm{Tr}\,(A_1 B_1 A_2 B_2 - A_2 B_1 A_1 B_2)$$

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$\Rightarrow$ both symmetries together generate $SU(4) \simeq SO(6)$, the R-symmetry of $N = 6$ supersymmetry

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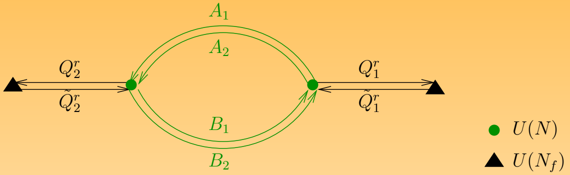

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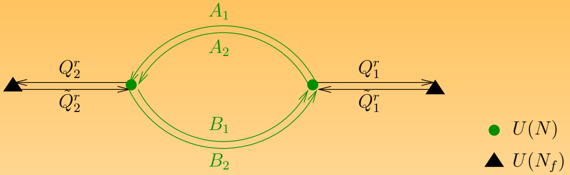

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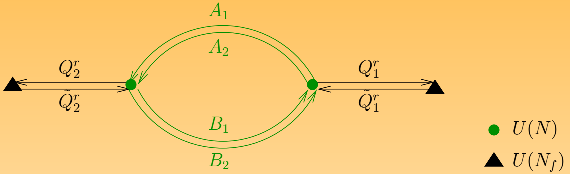

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- again, integrate out $\Phi_{1,2}$, get

$$\begin{aligned} & W_{\text{ABJM}} + W_{\text{flavour}} \\ &= \frac{4\pi}{k} \Big[\mathrm{Tr} \left(A_1 B_1 A_2 B_2 - A_2 B_1 A_1 B_2 \right) + \tilde{Q}_1 (A_1 B_1 + A_2 B_2) Q_1 \\ &\quad + \tilde{Q}_2 (B_1 A_1 + B_2 A_2) Q_2 + \frac{1}{2} (Q_1 \tilde{Q}_1)^2 - \frac{1}{2} (\tilde{Q}_2 Q_2)^2 \Big] \, . \end{aligned}$$

$N = 3$ supersymmetry

- define $SU(2)_R$ doublets of scalars

$$a^i_a = \begin{pmatrix} a_i \\ \bar{b}_i \end{pmatrix}, \quad \text{and} \quad q^1_a = \begin{pmatrix} q_1 \\ \bar{\tilde{q}}_1 \end{pmatrix}, \quad \text{and} \quad q^2_a = \begin{pmatrix} \tilde{q}_2 \\ \bar{q}_2 \end{pmatrix}$$

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$$\begin{aligned}
S = \frac{4\pi^2}{3k^2} \big[& q_a^1 \bar{q}_1^a q_b^1 \bar{q}_1^b q_c^1 \bar{q}_1^c + q_a^2 \bar{q}_2^a q_b^2 \bar{q}_2^b q_c^2 \bar{q}_2^c - 4 q_a^1 \bar{q}_1^b q_c^1 \bar{q}_1^a q_b^1 \bar{q}_1^c - 4 q_a^2 \bar{q}_2^b q_c^2 \bar{q}_2^a q_b^2 \bar{q}_2^c + \\
& + a_a^i \bar{a}_i^a a_b^j \bar{a}_j^b a_c^k \bar{a}_k^c + \bar{a}_i^a a_a^i \bar{a}_j^b a_b^j \bar{a}_k^c a_c^k + 4 a_a^i \bar{a}_j^b a_c^k \bar{a}_i^a a_b^j \bar{a}_k^c - 6 a_a^i \bar{a}_j^b a_b^j \bar{a}_i^a a_c^k \bar{a}_k^c \\
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& + 9 a_a^i \bar{a}_i^a q_b^1 \bar{q}_1^b q_c^1 \bar{q}_1^c + 9 \bar{a}_i^a a_a^i \bar{q}_2^b q_b^2 \bar{q}_2^c q_c^2 - 6 a_a^i \bar{a}_i^a q_b^1 \bar{q}_1^c q_c^1 \bar{q}_1^b - 6 \bar{a}_i^a a_a^i \bar{q}_2^b q_c^2 \bar{q}_2^c q_b^2 \\
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& - 6 \bar{a}_i^a q_b^1 \bar{q}_1^b a_a^i \bar{q}_2^c q_c^2 + 12 \bar{a}_i^a q_b^1 \bar{q}_1^c a_a^i \bar{q}_2^b q_c^2 + 12 \epsilon_{ij} \epsilon^{kl} a_c^i \bar{a}_k^b a_a^j \bar{a}_l^c q_b^1 \bar{q}_1^a \\
& + 12 \epsilon^{ij} \epsilon_{kl} \bar{a}_i^c a_b^k \bar{a}_j^a a_c^l \bar{q}_2^b q_a^2 \big] ,
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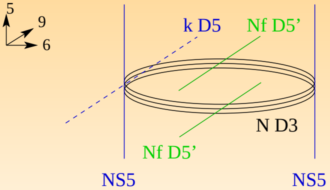
- $SU(2)_R$ commutes with $SU(2)_D$ $\longrightarrow$ only $N = 3$ susy

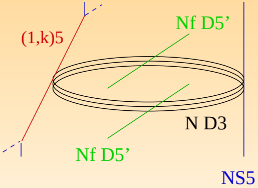
Type IIB brane setup

setup:

N	D3-branes	0126		
2	NS5-branes	012345	$\longrightarrow$	$(1,\pm k)$5-brane
k	D5-branes	012349		$012[3,7]_\theta[4,8]_\theta[5,9]_\theta$
$2N_f$	D5$'$-branes	012789		$(\tan\theta = k)$



before the web deformation
$\mathcal{N} = 2$ supersymmetry



web-deformed setup
$\mathcal{N} = 3$ supersymmetry

T-dual type IIA setup & lift to M-theory

<u>T-dual type IIA setup</u>:

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1	KKM	012345
1	$(1, \pm k)$6- "brane"	$0126[3,7]_\theta[4,8]_\theta[5,9]_\theta$
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$\Rightarrow$ N M2-branes at a toric hyperkähler geometry

- T^2 fibration over $\mathbb{R}^6$
- compatible with hyperkähler structure
- $sp(2)$ holonomy
- preserves $3/16$ susy of 11d sugra

Hypertoric geometry $\mathcal{M}_8$ with $SO(4)$ isometry

M-theory

- N M2-branes at a toric hyperkähler geometry $\mathcal{M}_8$

$$ds^2_{\mathcal{M}_8} = U_{ij} d\vec{x}^i \cdot d\vec{x}^j + U^{ij}(d\varphi_i + A_i)(d\varphi_j + A_j)$$

$$A_i = d\vec{x}^j \cdot \vec{\omega}_{ji} = dx^j_a \omega^a_{ji} \,, \quad \partial_{x^j_a} \omega^b_{ki} - \partial_{x^k_b} \omega^a_{ji} = \epsilon^{abc} \partial_{x^j_c} U_{ki}$$

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$$ds^2_{\mathcal{M}_8} = U_{ij} d\vec{x}^i \cdot d\vec{x}^j + U^{ij}(d\varphi_i + A_i)(d\varphi_j + A_j)$$

$$A_i = d\vec{x}^j \cdot \vec{\omega}_{ji} = dx^j_a \omega^a_{ji}\,, \quad \partial_{x^j_a}\omega^b_{ki} - \partial_{x^k_b}\omega^a_{ji} = \epsilon^{abc}\partial_{x^j_c} U_{ki}$$

- non-diagonalizable U_{ij} matrix:

$$U = \mathbf{1} + \begin{pmatrix} h_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} h_2 & kh_2 \\ kh_2 & k^2 h_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & N_f^2 h_3 \end{pmatrix}$$

$$h_1 = \frac{1}{2|\vec{x}_1|}\,, \quad h_2 = \frac{1}{2|\vec{x}_1 + k\vec{x}_2|}\,, \quad h_3 = \frac{1}{|N_f \vec{x}_2|}$$

Hypertoric geometry $\mathcal{M}_8$ with $SO(4)$ isometry

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- explicit solution for A_i (near-core region):

$$A_1 = \frac{(x^2_{12} dx^1_1 - x^1_{12} dx^2_1) + k(x^2_{12} dx^1_2 - x^1_{12} dx^2_2)}{2|\vec{x}_{12}|(|\vec{x}_{12}| + x^3_{12})} + \frac{x^2_1 dx^1_1 - x^1_1 dx^2_1}{2|\vec{x}_1|(|\vec{x}_1| + x^3_1)} \,,$$

$$A_2 = \frac{k(x^2_{12} dx^1_1 - x^1_{12} dx^2_1) + k^2(x^2_{12} dx^1_2 - x^1_{12} dx^2_2)}{2|\vec{x}_{12}|(|\vec{x}_{12}| + x^3_{12})} + \frac{N_f(x^2_2 dx^1_2 - x^1_2 dx^2_2)}{|\vec{x}_2|(|\vec{x}_2| + x^3_2)} \qquad (\vec{x}_{12} \equiv \vec{x}_1 + k\vec{x}_2)$$

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- $SO(3) \simeq SU(2)$ acts "diagonally" on $\vec{x}_1$ and $\vec{x}_2$

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- probe limit: small number of D6-branes, neglect backreaction on the $AdS_4 \times \mathbb{CP}^3$ near-horizon-geometry of ABJM model

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- need to show that embedding is stable and supersymmetric, i.e. $\mathbb{RP}^3$ is a special Lagrangian three-cycle inside $\mathbb{CP}^3$ (note: $\mathbb{CP}^3$ is not CY)

Embedding of the D6-branes

- Fubini-Study metric:

$$ds^2_{\mathbb{CP}^3} = \frac{d\bar{\zeta}_\alpha d\zeta^\alpha}{(1+\bar{\zeta}_\gamma\zeta^\gamma)^2} + \frac{\zeta^\alpha\bar{\zeta}_\beta d\bar{\zeta}_\alpha d\zeta^\beta}{(1+\bar{\zeta}_\gamma\zeta^\gamma)^4}$$

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- D6 wraps $\mathbb{RP}^3$

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- applications: fractional quantum Hall effect   Takayanagi *et al* 2009