Lepton Asymmetries and Their Evolution in E6SSM

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PASCOS, DESY, Hamburg, July 9 2009

Outline

The Baryon Asymmetry of Universe

Matter is dominant over anti-matter in the present Universe

WMAP's result:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10}$$

need to be explained.....

- Ingredients in Leptogenesis
  - Majorana RH neutrino mass (violating L)
    + Sphaleron process (violating $B + L$, conserving $B - L$)
  - Complexity of Yukawa couplings
  - Out-of-thermal processes
- The Lagrangian of SM + RH neutrinos

$$\mathcal{L} = \mathcal{L}_{SM} + h_{ik} \bar{N}_{Ri} \ell_{Lk} H - \frac{1}{2} M_{Nij} N_{Ri} N_{Rj} + h.c.$$

- Seesaw Mechanism

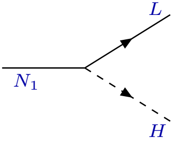
$$m_\nu = hh^T \frac{v^2}{M_R}$$

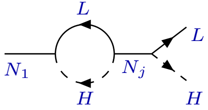
$M_R \sim 10^{14}\, GeV$ to explain $m_\nu \sim 10^{-1}\, eV$ [Mohapatra. 1980]

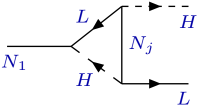
CP Asymmetry in Leptogenesis

- The Majorana nature of RH neutrino allows it to decay into both lepton and anti-lepton. The ratio is the same at tree level, but small differences arise at loop level [Yanagida. 86]

$$\epsilon_{i,\ell_k} \equiv \frac{\Gamma_{N_i \to \ell_k + H} - \Gamma_{N_i \to \bar{\ell}_k + H^*}}{\Gamma_{total}}$$







- We are interested in the asymmetry induced by the lightest RH neutrino ϵ_1. In non-susy models,

$$\epsilon_{1,\ell_k} = -\frac{1}{8\pi} \sum_{j=2,3} \frac{\text{Im}\left[(h^\dagger h)_{1j} h^\dagger_{1k} h_{kj}\right]}{(h^\dagger h)_{11}} \left[f_V\left(\frac{M_j^2}{M_1^2}\right) + f_S\left(\frac{M_j^2}{M_1^2}\right)\right]$$

where $f_V(x) = \sqrt{x}\left[-1 + (x+1)\ln\left(1 + \frac{1}{x}\right)\right]$, $f_S(x) = \frac{\sqrt{x}}{x-1}$.

- In the case of $M_1 \ll M_{2,3}$, it can be written as

$$\epsilon_{1,\,\ell_k} = -\frac{1}{4\pi} \sum_{j=2,3} \frac{\text{Im}\left[(h^\dagger h)_{1j} h^\dagger_{1k} h_{kj}\right]}{(h^\dagger h)_{11}} \frac{M_1}{M_j}$$

however, constrained by ν mass.

$\mathrm{E_6SSM}$

Exceptional Supersymmetric Standard Model ($\mathrm{E_6SSM}$) [King. 06] is based on

$$SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N \,,$$

a subgroup of E_6, where

$$E_6 \to SO(10) \times U(1)_\chi, \qquad SO(10) \to SU(5) \times U(1)_\psi$$

$$U(1)_N = \frac{1}{4}U(1)_\chi + \frac{\sqrt{15}}{4}U(1)_\psi$$

Particle content of $\mathrm{E_6SSM}$

- 3 generations of the $\mathbf{27}$ fundamental representation of E_6. $\Rightarrow$ anomalies cancelation.

  $$Q_{Li} \quad L_{Li} \quad u_{Ri} \quad d_{Ri} \quad e_{Ri} \quad N_i \quad H_{1i} \quad H_{2i} \quad S_i \quad D_i \quad \bar{D}_i$$

  with i the family index.

- Besides $\mathbf{27}$, extra fields with one generation $\Rightarrow$ gauge unification

  $$L_4, \overline{L_4}$$

E_6SSM

- Z_2^H symmetry $\Rightarrow$ suppress the proton decay and *FCNC*
  - Odd for all fields, except $H_d \equiv H_{1,3}$, $H_u \equiv H_{2,3}$ and $S \equiv S_3$

$$\begin{aligned} W_{\mathrm{E_6SSM}} \quad \simeq \quad & \lambda_i S(H_{1i}H_{2i}) + \kappa_i S(D_i\overline{D}_i) + f_{\alpha\beta}(H_d H_{2\alpha})S_\beta + \tilde{f}_{\alpha\beta}(H_{1\alpha}H_u)S_\beta \\ & + h^U_{ij}(H_u Q_i)u^c_j + h^D_{ij}(H_d Q_i)d^c_j + h^E_{ij}(H_d L_i)e^c_j + h^N_{ij}(H_u L_i)N^c_j \\ & + \frac{1}{2}M_{ij}N^c_i N^c_j + \mu'(L_4\overline{L}_4) + h^E_{4j}(H_d L_4)e^c_j + h^N_{4j}(H_u L_4)N^c_j \end{aligned}$$

L_4 has one lepton number.

- The breaking of Z_2^H gives extra terms in the superpotential:

$$W_N = \xi_{\alpha ij}(H_{2\alpha}L_i)N^c_j + \xi_{\alpha 4j}(H_{2\alpha}L_4)N^c_j$$

  - Model I, D - diquark

$$W_1 = g^Q_{ijk}D_i\,(Q_j Q_k) + g^q_{ijk}\overline{D}_i d^c_j u^c_k$$

  - Model II, D - leptoquark

$$W_2 = g^N_{kij}D_k d^c_i N^c_j + g^E_{kij}D_k u^c_i e^c_j + g^D_{ijk}\,(Q_i L_j)\,\overline{D}_k$$

- The superpotential of interactions of RH neutrinos can be rewritten with compact notations:

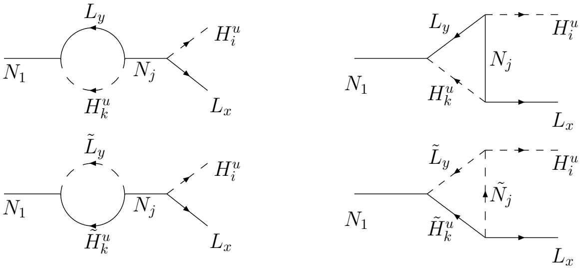
$$W_N = h^N_{kxj}(H^u_k L_x)N^c_j + g^N_{kij}D_k d^c_i N^c_j$$

where $x = 1, 2, 3, 4$, $k, i, j = 1, 2, 3$

CP Asymmetry in E_6SSM, Z_2^H conserved case

- New contributions to N_1 decay in the case of conserved Z_2^H :

$$N_1 \to L_x + H_k^u, \quad N_1 \to \widetilde{L}_x + \widetilde{H}_k^u, \qquad \widetilde{N}_1 \to \overline{L}_x + \overline{\widetilde{H}}{}_k^u, \quad \widetilde{N}_1 \to \widetilde{L}_x + H_k^u$$



Here $x, y = 1, 2, 3, 4$ and $i, k = 3$, representing the Higgs in the SM.

- Z_2^H conserved case, distributed in L_x and ordinary leptons respectively

$$\varepsilon^3_{1,\,\ell_x} \simeq -\frac{3}{8\pi} \sum_{j=2,3} \frac{\mathrm{Im}\left[(h^{N\dagger} h^N)_{1j} h^{N*}_{x1} h^N_{xj}\right]}{(h^{N\dagger} h^N)_{11}} \frac{M_1}{M_j}$$

[S. King, RL, D. Miller, R. Nevzorov. 08]

CP Asymmetry in $\mathrm{E}_6\mathrm{SSM}$, $\mathrm{Z}_2^{\mathrm{H}}$ violating case (model I)

- In Z_2^H symmetry violating case (Model I), the 1st and 2nd generations of Higgs are considered ($k = 1, 2, 3$).

$$N_1 \to L_x + H_k^u, \qquad N_1 \to \widetilde{L}_x + \widetilde{H}_k^u, \qquad \widetilde{N}_1 \to \overline{L}_x + \overline{\widetilde{H}}_k^u, \qquad \widetilde{N}_1 \to \widetilde{L}_x + H_k^u$$

- Similarly, one-loop calculation gives CP asymmetries of different final states:

$$\varepsilon^k_{1,\,\ell_x} = \varepsilon^k_{1,\,\widetilde{\ell}_x} = \varepsilon^k_{\widetilde{1},\,\ell_x} = \varepsilon^k_{\widetilde{1},\,\widetilde{\ell}_x} = \frac{1}{8\pi A_1} \sum_{j=2,3} \mathrm{Im}\Big\{ 2\, A_j h^{N*}_{kx1} h^N_{kxj} \frac{M_1}{M_j} + \sum_{m,\,y} h^{N*}_{my1} h^N_{mxj} h^N_{kyj} h^{N*}_{kx1} \frac{M_1}{M_j} \Big\}$$

where $A_j = \sum_{m,y} h^{N*}_{my1} h^N_{myj}$ and $x, y = 1, 2, 3, 4,\; i, k = 1, 2, 3$.

Lepton Asymmetry in Constrained Sequential Dominance of See-saw model

- In the basis where RH neutrino mass matrix is diagonal and LH neutrinos in ew eigenstates.

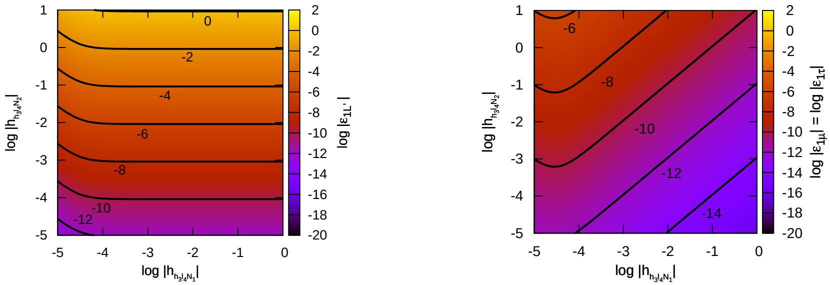
$$M_N = \begin{pmatrix} M_A & 0 & 0 \\ 0 & M_B & 0 \\ 0 & 0 & M_C \end{pmatrix} \qquad h^N = \begin{pmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{pmatrix}$$

- It is natural to assume $\frac{|A_i A_j|}{M_A} \gg \frac{|B_i B_j|}{M_B} \gg \frac{|C_i C_j|}{M_C}$, in the case of strong-hierarchy of RH neutrino masses. And $|A_1| \ll |A_{2,3}|$
- The LH neutrino masses from see-saw model:

$$m_3 \simeq \frac{(|A_2|^2 + |A_3|^2)v^2}{M_A}, \quad m_2 \simeq \frac{|B_1|^2 v^2}{s_{12}^2 M_B}, \quad m_1 \simeq \mathcal{O}(\frac{|C|^2 v^2}{M_C})$$

  where v is the vev of Higgs potential, and $s_{12} \equiv \sin\theta_{12} \sim \sqrt{1/3}$
- For $m_1 \ll m_2 \ll m_3$, LH neutrino masses can be determined: $m_1 \sim 0$, $m_2 \simeq 8.7 \times 10^{-3}\, eV$, $m_3 \simeq 4.9 \times 10^{-2} eV$. Then Yukawa couplings depend on RH neutrino mass $M_{A,B}$.

Lepton Asymmetry with L_4



- Lepton asymmetries versus leptoquark couplings in E_6SSM Z_2 conserved case
- We take $M_1 = 10^6\,GeV$, and $M_2 = 10M_1$.
- Lepton asymmetries are expected to be $\sim 10^{-4} - 10^{-6}$, to generate right amount of baryon asymmetry.

Flavour Independent Boltzman Equations

The evolution of flavoured lepton asymmetries is described by a set of Boltzmann Equations.

$$\frac{dY_{N_1}}{dz} = \frac{-z}{sH(M_1)}(\gamma_D + \gamma_{S,\Delta L=1,2})\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right),$$

$$\frac{dY_{\Delta_\alpha}}{dz} = \frac{-z}{sH(M_1)}\left[\epsilon_\alpha(\gamma_D + \gamma_{S,\Delta L=1})\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right) - (\frac{\gamma^\alpha_D}{2} + \gamma^\alpha_{W,\Delta L=1,2})\frac{Y_{\Delta\alpha}}{Y^{eq}_\ell}\right].$$

- $Y_{\Delta_\alpha} \equiv Y_{B/3} - Y_{L_\alpha}$ and Y_{N_1} are the density of $B - L_\alpha$ and N_1 respectively.
- γ_D, $\gamma_{S,\Delta L=1,2}$, $\gamma_{W,\Delta L=1,2}$ are the thermally averaged reaction density for RH neutrino decay, $\Delta L = 1, 2$ scatterings and wash-out processes.
- $z \equiv M_1/T$ is a dimensionless parameter.

Number density of particle in equilibrium:

$$Y^{eq}_\ell = \frac{45}{\pi^4 g_*}, \qquad Y^{eq}_{N_1} = \frac{45}{2\pi^4 g_*} z^2 K_2(z).$$

where $K_2(z)$ is the modified Bessel function.

Numerical result of Boltzmann equations without scattering terms[1]

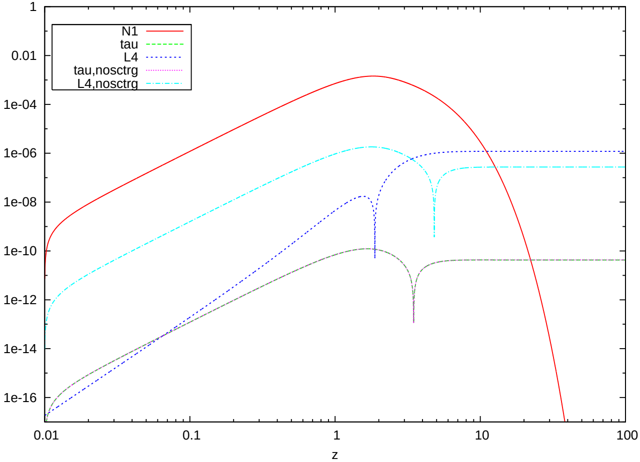


Figure: Evolution of N_1, τ and $Ł_4$, when $h_{41} = 10^5$, $h_{42} = 1$

[1] work in progress with Stefan Antusch, Steve King, David Miller, and Roman Nevzorov

Why Flavoured Boltzmann Equations?

Spectator Processes involved in generating Lepton/Baryon Asymmetries.

- $SU(2)$ electro-weak instanton process (left-handed leptons $\leftrightarrow$ left-handed quarks)
- $SU(3)$ QCD instanton process (left-handed quarks $\leftrightarrow$ right-handed quarks)
- Yukawa interactions ($\ell_i \leftrightarrow H + e_j$, $u_i \leftrightarrow H + Q_i$, $d_i \leftrightarrow H + Q_i$)

- The Yukawa interactions in thermal equilibrium (if $\gamma > H(z)$) for some lepton flavours *e.g.* τ, at some temperature, whereas some may not *e.g.* e

Features of Spectator processes

- Much faster than (inverse) decays and scattering
- Conserve $B - L$
- Relavant components in certain ratios

Uni-flavoured Boltzmann Equations (in the SM+N)

Number density and chemical potential:

$$n_X - n_{\bar{X}} = \frac{g_X T^3}{6} \cdot \mu_X / T \quad \text{for fermions}$$

$$n_X - n_{\bar{X}} = \frac{g_X T^3}{6} \cdot 2\mu_X / T \quad \text{for bosons}\,.$$

- The Yukawa interactions in equilibrium ($Q \leftrightarrow u + H^*$, $Q \leftrightarrow d + H$ and $\ell \leftrightarrow e + H$) gives

$$\mu_Q - \mu_u + \mu_H = 0\,,$$
$$\mu_Q - \mu_d - \mu_H = 0\,,$$
$$\mu_\ell - \mu_e - \mu_H = 0\,.$$

- The electroweak sphaleron process conserves $B/3 + L$

$$3\mu_Q + \mu_\ell = 0\,,$$

- The hypercharge neutrality requires

$$3\mu_Q + 4\mu_u - 2\mu_d - 3\mu_\ell - 2\mu_e + 2\mu_H = 0\,.$$

Uni-flavoured Boltzmann Equations (in the SM+N)

In the temperature range $T \lesssim 10^9\ \mathrm{GeV}$, all Yukawa couplings are in equilibrium,

$$\mu_Q = -\frac{\mu_\ell}{3}; \quad \mu_H = \frac{7\mu_\ell}{15}; \quad \mu_c = \frac{2\mu_\ell}{15}; \mu_s = -\frac{4\mu_\ell}{5}; \quad \mu_\mu = \frac{8\mu_\ell}{15}\ .$$

We define $\Upsilon \equiv \sum_{\text{flavour}} (e - 2\,Q - u - d)$, the "inactive" component in $B - L$.
Spectator processes keep Υ and ℓ_α in ratio $c_\Upsilon = \frac{Y_{\ell_\alpha}}{Y_\Upsilon} = 5/68$.

The Boltzmann Equations with spectator processes

$$\begin{aligned}
\frac{dY_{N_1}}{dz} &= \frac{-z}{sH(M_1)}(\gamma_D + \gamma_{S,\Delta L=1,2})\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\ , \\
\frac{dY_{\ell_\alpha}}{dz} &= \frac{z}{sH(M_1)}\left[\epsilon_\alpha(\gamma_D + \gamma_{S,\Delta L=1})\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right) - (\frac{\gamma_D^\alpha}{2} + \gamma_{W,\Delta L=1,2}^\alpha)\frac{Y_{\ell_\alpha}}{Y_\ell^{eq}}\right] \\
&\quad + \Theta\left(c_\Upsilon Y_\Upsilon + \sum_{\beta\neq\alpha} Y_{\ell_\beta} - N_f\, Y_{\ell_\alpha}\right)\ , \\
\frac{dY_\Upsilon}{dz} &= \Theta\left(\sum_\beta Y_{\ell_\beta} - N_f\, c_\Upsilon\, Y_\Upsilon\right)\ .
\end{aligned}$$

wehre N_f is the number of flavours. And Θ is a "large" number.

Uni-flavoured Boltzmann Equations (in the SM+N) – Numerical Result

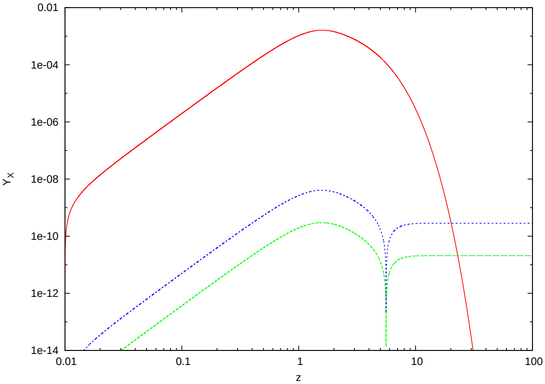


Figure: The evolution of N_1 abundance (the red line) left-handed lepton asymmetries Y_{ℓ_α} (the green line) and Υ total abundance (the blue line), for $M_1 = 10^8\ \mathrm{GeV}$.

Summary

- Leptogenesis provides an elegant explanation to BAU.
- $\mathrm{E}_6\mathrm{SSM}$
- New contribution of *CP* asymmetry from $\mathrm{E}_6\mathrm{SSM}$
- Result in the Sequential Dominance scenario
- Lepton Asymmetries can be enhanced drastically.
- Boltzmann equations (flavour independent/flavoured) for Baryon asymmetry.