Quantum Mechanics of Leptogenesis

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Towards

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Baryogenesis

The Origin of Matter

- universe is made of matter
  $\Rightarrow$ baryon asymmetry $\eta \approx 10^{-10}$
- inflationary cosmology: η created after inflation
- requires three Sakharov conditions
  - baryon number violation
  - *C* and *CP* violation
  - deviation from thermal equilibrium

Leptogenesis

Basic Idea

- matter-antimatter asymmetry is created in the leptonic sector by decay of heavy Majorana neutrinos N

  Fukugita, Yanagida, Luty, Covi, Vissani, Roulet, Buchmüller, Plümacher etc

- asymmetry is partly transferred to baryons via sphaleron processes

- large mass of N allows to understand smallness of neutrino masses via see saw mechanism

  $\Rightarrow$ one additional ingredient solves two puzzles!

The Problem

- usual treatment: Boltzmann equations
- BUT : quantum and memory effects can be relevant
  e.g. Buchmüller, Fredenhagen, Di Bari, Riotto, Simone, Raffelt, Blanchet etc
- quantum Boltzmann equations have been used
- full quantum mechanical treatment is desirable!
  $\Rightarrow$ Kadanoff-Baym equations

Is QM Treatment possible?

- coupling extremely weak $\Rightarrow$ perturbative
- all particles except N in equilibrium
- backreaction can be neglected

'Weak coupling to a thermal bath'

Our Approach

Steps...

1. **use scalar model for qualitative understanding**
   Anisimov/Buchmüller/M.D./Mendizabal 2008
2. **minimal scenario $M_1 \ll M_{2,3}$ and $T = const$**
3. **include Hubble expansion and $T(t)$**

Kadanoff-Baym Equations

- initial value problem for $\rho(t)\ldots$
- $\ldots$or for correlation functions $\langle\ldots\rangle = \mathrm{Tr}(\rho\ldots)$
- KB equations contain full quantum mechanics

Consider correlation functions rather than number densities!

The Baryon Asymmetry

Spectral and Statistical Propagators

$$\Delta^+(x_1,x_2) = \frac{1}{2}\langle\{\Phi(x_1),\Phi(x_2)\}\rangle$$
$$\Delta^-(x_1,x_2) = i\langle[\Phi(x_1),\Phi(x_2)]\rangle$$
$$S^+(x_1,x_2) = \frac{1}{2}\langle[\Psi(x_1),\bar{\Psi}(x_2)]\rangle$$
$$S^-(x_1,x_2) = i\langle\{\Psi(x_1),\bar{\Psi}(x_2)\}\rangle$$

Use total charge operator

$$\begin{aligned} \eta \propto Q(t) &\sim \int d^3\mathbf{x}\mathrm{tr}\left(\gamma^0\Psi(x_1)\bar{\Psi}(x_2)\right)|_{t_1=t_2=t} \\ &\sim S^+_{\mathbf{q}=0}(t_1=t_2=t) \end{aligned} \quad (1)$$

Kadanoff-Baym Equations

$$
\begin{aligned}
(\Box_1 + m^2)\Delta^-(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt'\Pi^-(x_1,x')\Delta^-(x',x_2) \\
(\Box_1 + m^2)\Delta^+(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_i}^{t_1} dt'\Pi^-(x_1,x')\Delta^+(x',x_2) \\
&\quad + \int d^3\mathbf{x}' \int_{t_i}^{t_2} dt'\Pi^+(x_1,x')\Delta^-(x',x_2)
\end{aligned}
$$

$$
\begin{aligned}
-(i\partial\!\!\!/_1 - m)S^-(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_2}^{t_1} dt'\Sigma^-(x_1,x')S^-(x',x_2) \\
-(i\partial\!\!\!/_1 - m)S^+(x_1,x_2) &= -\int d^3\mathbf{x}' \int_{t_i}^{t_1} dt'\Sigma^-(x_1,x')S^+(x',x_2) \\
&\quad + \int d^3\mathbf{x}' \int_{t_i}^{t_2} dt'\Sigma^+(x_1,x')S^-(x',x_2)
\end{aligned}
$$

Solutions for Scalars

$$\Delta^-(t_1 - t_2) = i \int \frac{d\omega}{2\pi} \frac{-2e^{-i\omega(t_1-t_2)}\mathrm{Im}\Pi^R_{\mathbf{q}}(\omega) - \omega\epsilon}{\left(\omega^2 - \omega_{\mathbf{q}}^2 - \mathrm{Re}\Pi^R_{\mathbf{q}}(\omega)\right)^2 + \left(\mathrm{Im}\Pi^R_{\mathbf{q}}(\omega) + \omega\epsilon\right)^2}$$

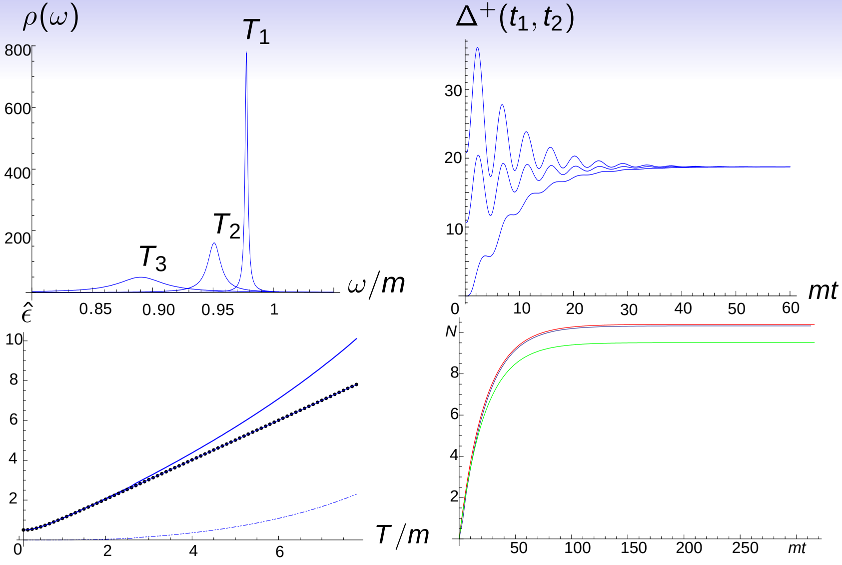
$$\begin{aligned}
\Delta^+(t_1, t_2) &= \Delta^+_{,\mathrm{in}}\dot{\Delta}^-(t_1)\dot{\Delta}^-(t_2) + \ddot{\Delta}^+_{,\mathrm{in}}\Delta^-(t_1)\Delta^-(t_2) \\
&+ \dot{\Delta}^+_{;\mathrm{in}}\left(\dot{\Delta}^-(t_1)\Delta^-(t_2) + \Delta^-(t_1)\dot{\Delta}^-(t_2)\right) \\
&+ \int_0^{t_1} dt' \int_0^{t_2} dt'' \Delta^-(t_1 - t')\Pi^+(t' - t'')\Delta^-(t'' - t_2)
\end{aligned}$$

Properties of the Plasma

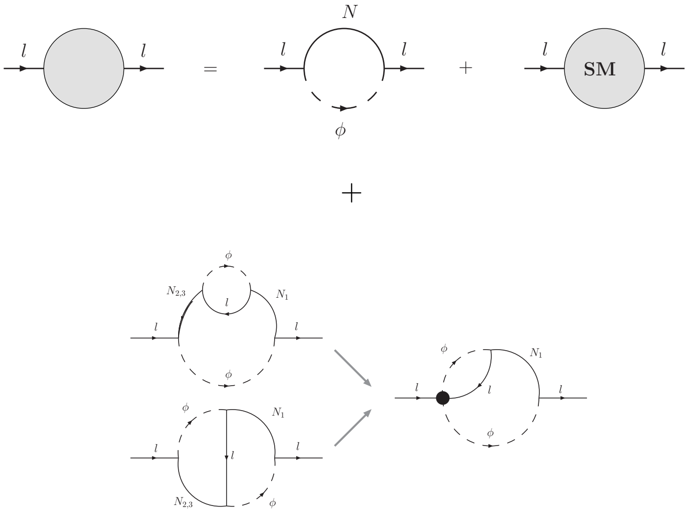
Three regimes

$\mathrm{Re}\Pi^R$ gives thermal mass , $\mathrm{Im}\Pi^R$ decay width to resonance

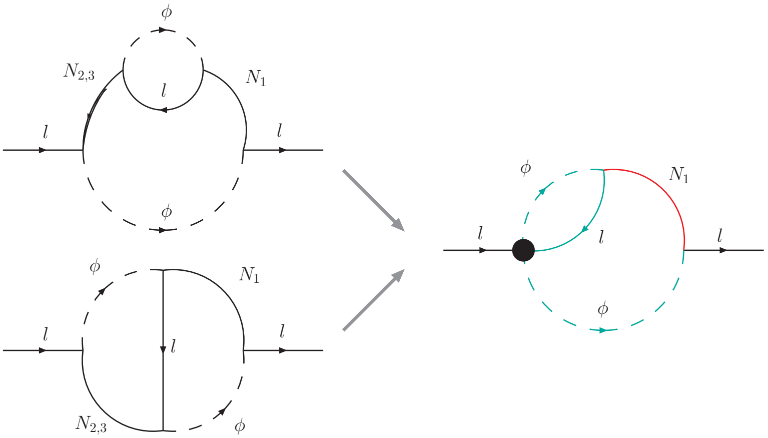
1. $|\mathrm{Re}\Pi|, |\mathrm{Im}\Pi| \ll m^2$
   (quasi)particle behaviour
2. $|\mathrm{Re}\Pi| \approx m^2, |\mathrm{Im}\Pi| \ll m^2$
   single resonance kinematically behaves like quasiparticle
   but total energy recieves vacuum contribution
3. $|\mathrm{Re}\Pi|, |\mathrm{Im}\Pi| \approx m^2$
   particle interpretation and Boltzmann equations break down


ρ(ω)
T1
T2
T3
800
600
400
200
0.85
0.90
0.95
1
ω/m
Δ+(t1, t2)
30
20
10
0
10
20
30
40
50
60
mt
ε̂
10
8
6
4
2
0
2
4
6
T/m
N
8
6
4
2
50
100
150
200
250
mt

Lepton Self-Energy



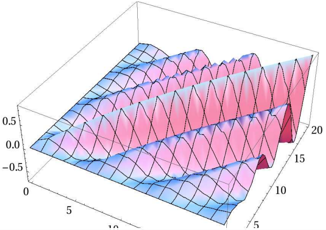
CP-violating Part $\delta\Sigma$

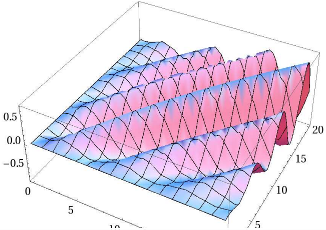


Majorana Propagator

$$G^-(t_1 - t_2) = e^{-\Gamma/2|t_1-t_2|}\Big(i\gamma^0 \cos(\omega_{\mathbf{q}}(t_1 - t_2)) - \frac{\boldsymbol{\gamma}\mathbf{q} - M}{\omega_{\mathbf{q}}} \sin(\omega_{\mathbf{q}}(t_1 - t_2))\Big) C^{-1}$$

$$G^+(t_1, t_2) = \frac{\tanh\left(\beta\omega_{\mathbf{q}}/2\right)}{2\omega_{\mathbf{q}}} \left(e^{-\Gamma/2|t_1-t_2|} - e^{-\Gamma/2(t_1+t_2)}\right) \times \Big((M - \boldsymbol{\gamma}\mathbf{q}) \cos(\omega_{\mathbf{q}}(t_1 - t_2)) - i\gamma^0\omega_{\mathbf{q}} \sin(\omega_{\mathbf{q}}(t_1 - t_2))\Big) C^{-1}$$





Conclusions

- behaviour of the hot plasma can deviate significantly from quasiparticle picture
- quantum and memory effects could be important for leptogenesis
- full understanding of the quantum dynamics of the heavy Majorana neutrino has been achieved
- the results are curently used to compute $\eta(t)$ in a simple model