Renormalization techniques for nonequilibrium quantum fields

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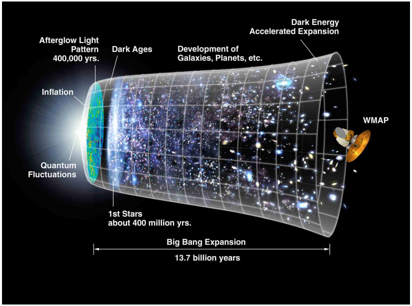
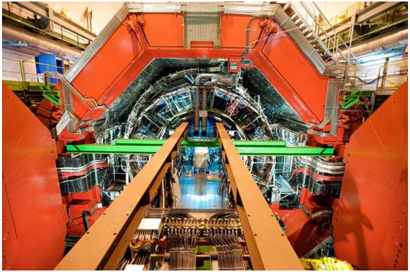
based on 0904.3600 with Markus Michael Müller; Urko Reinosa

Nonequilibrium dynamics at high energy



Heavy Ion Collisions

- LHC: ALICE
- RHIC



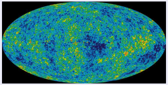
Early universe

- Reheating after Inflation
- Baryogenesis
- ...

Nonequilibrium dynamics at high energy

Baryogenesis: three Sakharov conditions

- baryon number violation
- CP violation
- deviation from thermal equilibrium

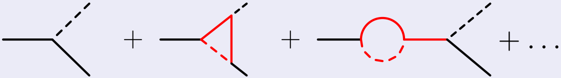


$\eta_{10} = (n_b - n_{\bar{b}})/(s \cdot 10^{-10})$

$4.7 < \eta_{10} < 6.5$ (95% CL)

Leptogenesis: decay of heavy right-handed neutrino N_i

$\mathcal{M}_{N_i \to l_\alpha h^\dagger} =$



CP violation in decay described by loop process

Quantum nonequilibrium effects ?

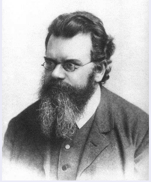
Usual description: Boltzmann equations

Outline

Renormalization techniques for nonequilibrium quantum fields

- The classical approach: Boltzmann
- Nonequilibrium quantum field theory: Kadanoff-Baym
- Renormalization techniques: Non-Gaussian initial states

Boltzmann equation for one-particle distribution functions

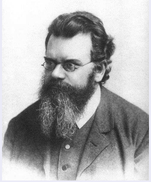


$$p^\alpha \mathcal{D}_\alpha f_\psi(t, \mathbf{x}, \mathbf{p}) = \int d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots$$
$$\times (2\pi)^4 \delta(p_\psi + p_a + p_b \ldots - p_i - p_j)$$
$$\times \Big[|\mathcal{M}|^2_{i+j\ldots\to\psi+a+b\ldots} f_i f_j \cdots (1 \pm f_a)(1 \pm f_b)(1 \pm f_\psi)$$
$$- |\mathcal{M}|^2_{\psi+a+b\ldots\to i+j\ldots} f_a f_b f_\psi \cdots (1 \pm f_i)(1 \pm f_j) \ldots \Big]$$

$|\mathcal{M}|^2$: microscopic interactions, off-shell processes
$f_\psi(t, \mathbf{x}, \mathbf{p})$: macroscopic propagation of on-shell particles

The classical approach: Boltzmann equations

Boltzmann equation for one-particle distribution functions



$$p^\alpha \mathcal{D}_\alpha f_\psi(t, \mathbf{x}, \mathbf{p}) \;=\; \int d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots$$

$$\times\, (2\pi)^4 \delta(p_\psi + p_a + p_b \ldots - p_i - p_j)$$

$$\times \Big[|\mathcal{M}|^2_{i+j\ldots\to\psi+a+b\ldots}\; f_i f_j \cdots (1 \pm f_a)(1 \pm f_b)(1 \pm f_\psi)$$

$$- |\mathcal{M}|^2_{\psi+a+b\ldots\to i+j\ldots}\; f_a f_b f_\psi \cdots (1 \pm f_i)(1 \pm f_j) \ldots \Big]$$

$|\mathcal{M}|^2$: microscopic interactions, off-shell processes

$f_\psi(t, \mathbf{x}, \mathbf{p})$: macroscopic propagation of on-shell particles

$$\Delta x_{interaction} \ll \lambda_{mfp}, \quad \lambda_{de-Broglie} \ll \lambda_{mfp}$$

$$1/M \ll 1/\Gamma, \quad 1/T \ll g^2/T$$

Corrections within Boltzmann picture

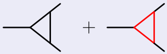
Bose-enhancement, Pauli-Blocking; kinetic (non-)equilibrium

- quantum statistical factors $1 \pm f_k$
- non-integrated Boltzmann equations

Hannsestad, Basbøll 06; Garayoa, Pastor, Pinto, Rius, Vives 09; Hahn-Woernle, Plümacher, Wong 09

Medium corrections

- medium correction to decay rates



$$\epsilon_i = \frac{\Gamma(N_i \to l h^\dagger) - \Gamma(N_i \to l^c h)}{\Gamma(N_i \to l h^\dagger) + \Gamma(N_i \to l^c h)} = \epsilon_i^{vac} + \delta\epsilon_i^{th}(T, \ldots)$$

- thermal masses

Giudice, Notari, Raidal, Riotto, Stumia 04; Covi, Rius, Roulet, Vissani 98;...

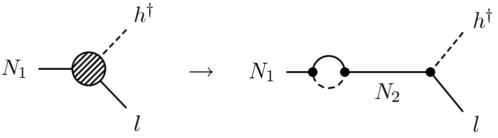
Flavour effects

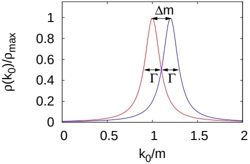
Nardi, Nir, Roulet, Racker 06; Adaba, Davidson, Ibarra, Josse-Micheaux, Losada, Riotto 06; Blanchet, diBari 06...

Limitations of the Boltzmann picture

- Spectral function $\neq$ quasi-particles

  $\Rightarrow$ Mixing fields in Resonant Leptogenesis *Pilaftsis, Underwood, ...*





$$\Delta x^2_{interaction} \sim \frac{1}{k_1^2 - M_2^2} \sim \frac{1}{M_1^2 - M_2^2}, \qquad \lambda^2_{mfp} \sim \frac{1}{\Gamma^2}$$

- Memory & Correlation effects, Higher Gradients
- Double Counting Problem(s) for real intermediate states *Pilaftsis,. . .*
- Find a controlled expansion

Going beyond the Boltzmann picture

$$\text{Statistical propagator} \quad G_F^{ij}(x,y) = \langle \Phi^i(x)\Phi^j(y) + \Phi^j(y)\Phi^i(x)\rangle/2$$

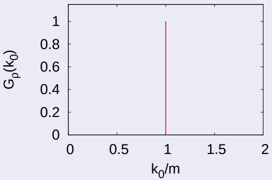
$$\text{Spectral function} \quad G_\rho^{ij}(x,y) = i\langle \Phi^i(x)\Phi^j(y) - \Phi^j(y)\Phi^i(x)\rangle$$

Going beyond the Boltzmann picture

Statistical propagator	$G_F^{ij}(x,y)$	$=$	$\langle \Phi^i(x)\Phi^j(y) + \Phi^j(y)\Phi^i(x)\rangle/2$
Spectral function	$G_\rho^{ij}(x,y)$	$=$	$i\langle \Phi^i(x)\Phi^j(y) - \Phi^j(y)\Phi^i(x)\rangle$

Boltzmann limit

- on-shell quasi-stable particles



$$G_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like KMS relation

$$G_F^{ij}(t,k) = \left(f_k^i(t) + \frac{1}{2}\right) G_\rho^{ij}(k)$$

Going beyond the Boltzmann picture

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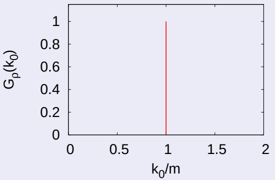

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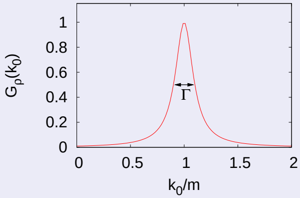
Propagation beyond Boltzmann

- spectrum with (thermal) width



$$G_\rho^{ij}(t,k) \sim \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2 \Gamma_i(t)^2} + \dots$$

- off-shell excitations

$$G_F^{ij}(t,k) = \begin{pmatrix} G_F^{11} & G_F^{12} \\ G_F^{21} & G_F^{22} \end{pmatrix}$$

Kadanoff-Baym equations

$$\left(\Box_x + m_i^2(x)\right) G_F^{ij}(x,y) = \int_0^{y^0} d^4z\, \Pi_F^{ik}(x,z) G_\rho^{kj}(z,y) - \int_0^{x^0} d^4z\, \Pi_\rho^{ik}(x,z) G_F^{kj}(z,y)$$

$$\left(\Box_x + m_i^2(x)\right) G_\rho^{ij}(x,y) = \int_{x_0}^{y^0} d^4z\, \Pi_\rho^{ik}(x,z) G_\rho^{kj}(z,y)$$

- Statistical propagator encodes time-evolution of the state
- Spectral function includes off-shell effects self-consistently
- Memory integrals

Nonequilibrium Quantum Field Theory

Obtained from stationarity condition of the 2PI effective action

$$\frac{\delta\Gamma[\phi, G]}{\delta G} = 0$$

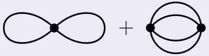
Cornwall, Jackiw, Tomboulis (1974)

Controlled approximation...

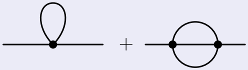
... by truncation of the 2PI functional $\Gamma_2[\phi, G]$

Example: Three-loop truncation in $\lambda\Phi^4$-theory (for $\langle\Phi\rangle = 0$)

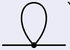
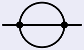
$$\Gamma_2[G] \quad =$$

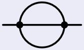


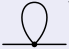
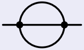
$$\Pi[G] = \frac{2i\delta\Gamma_2}{\delta G} \quad =$$



Kadanoff-Baym equations

$$\left(\Box_x + m^2 + \right.$$  $$\left.\right) G_F(x,y) = \int_0^{y^0} d^4z$$  $$G_\rho(z,y)$$

$$- \int_0^{x^0} d^4z$$  $$G_F(z,y)$$

$$\left(\Box_x + m^2 + \right.$$  $$\left.\right) G_\rho(x,y) = \int_{x_0}^{y^0} d^4z$$  $$G_\rho(z,y)$$

- Insert $\Pi[G] = \delta\Gamma_2/\delta G$ (e.g. 2PI-3loop 'setting-sun' approximation)
- Use $G(x,y) = G_F(x,y) - i/2\,\mathrm{sign}(x^0 - y^0)G_\rho(x,y)$

$\Rightarrow$ Closed set of self-consistent, non-perturbative equations

Nonequilibrium Quantum Field Theory

- Quantum thermalization

  Berges, Cox (2001); Berges (2002); Aarts, Berges (2002); Aarts, Resco (2003); Berges, Borsanyi, Serreau (2003); Juchem, Cassing, Greiner (2004); Arrizabalaga, Smit, Tranberg (2005); Lindner, Müller (2006); Gasenzer, Pawlowski (2008); . . .

- Prethermalization

  Berges, Borsanyi, Wetterich (2004)

- Parametric Resonance

  Berges, Serreau (2003)

- Nonequilibrium Instabilities

  Aarts, Tranberg (2007); Berges, Rothkopf, Schmidt (2008); Berges, Pruschke, Rothkopf (2009)

- Curved Spacetime

  Tranberg (2008); Hohenegger, Kartavtsev, Lindner (2008)

- Leptogenesis/Baryogenesis

  Buchmüller, Fredenhagen (2000); DeSimone, Riotto (2007); Anisimov, Buchmüller, Drewes, Mendizabal (2008)

Nonperturbative Renormalization

- 2PI Thermal field theory: established

  Hees, Knoll (2001, 2002); Blaizot, Iancu, Reinosa (2003); Berges, Borsanyi, Reinosa, Serreau (2004, 2005); Reinosa, Serreau (2006, 2007, 2009)

- 2PI Nonequilibrium field theory

  Borsanyi, Reinosa (2008); MG, Müller (2009)

Renormalization techniques for Kadanoff-Baym equations

Why renormalization?

- Numerical solutions use approximate renormalization
- Substantial cutoff-dependence
- Renormalization required for *quantitative* comparison with Boltzmann equations

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Problem

- Standard Kadanoff-Baym equations: Gaussian initial states
- Incompatible with renormalization

State of the Art: Gaussian initial state *e.g. Berges, Cox (2001)*

All n-point correlation functions vanish at $t = t_{init}$ for $n \geq 3$

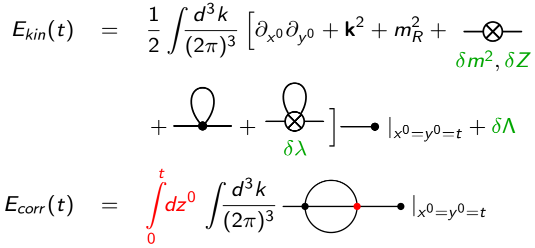
$$\alpha_n(x_1, \ldots, x_n) = 0 \quad \text{for } n \geq 3$$

Physical initial state *MG, Müller (2009)*

n-point correlation functions asymptotically agree with vacuum correlations at short distances [for $n \leq 4$]

$$\alpha_n(x_1, \ldots, x_n) = \alpha_n^{th}(x_1, \ldots, x_n) + \Delta\alpha_n(x_1, \ldots, x_n)$$

$$E_{total} = E_{kin}(t) + E_{corr}(t)$$



- $E_{corr}(t)$ contains divergences
- $E_{kin}(t)$ contains counterterms

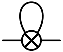
Why does the Gaussian initial state lead to singularities ?

$$E_{total} = E_{kin}(t) + E_{corr}(t)$$

$$E_{kin}(t) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\Big[\partial_{x^0}\partial_{y^0} + \mathbf{k}^2 + m_R^2 +$$

$$+$$

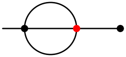
$$+$$



$$\Big]$$

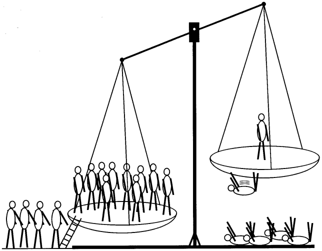
$$\Big|_{x^0=y^0=t} + \delta\Lambda$$

$$E_{corr}(t) = \int_0^t dz^0 \int \frac{d^3k}{(2\pi)^3}$$



$$\Big|_{x^0=y^0=t}$$

- $E_{corr}(t)$ contains divergences
- $E_{kin}(t)$ contains counterterms
- $E_{corr}(t)|_{t=0} = 0$ for Gaussian initial state
- $\Rightarrow$ unbalanced divergence at $t = 0$



Non-Gaussian initial states

Non-Gaussian density matrix at initial time — Calzetta, Hu (1988)

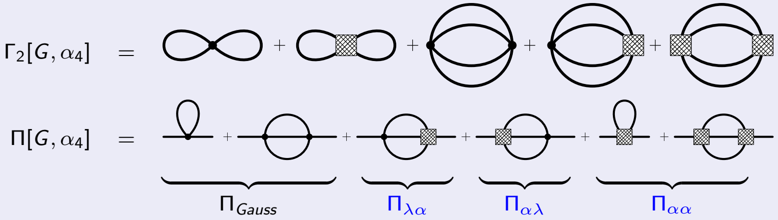
$$\langle \varphi_+ | \rho_{init} | \varphi_- \rangle = \exp\left(i \sum_{n=0}^{\infty} \int_{\mathcal{C}} d^4x_1 \ldots d^4x_n \, \alpha_n(x_1, \ldots, x_n) \varphi(x_1) \cdots \varphi(x_n) \right)$$

Effective non-local vertices

$$\alpha_n(x_1, \ldots, x_n) \sim \delta(x_1^0 - t_{init}) \cdots \delta(x_n^0 - t_{init})$$

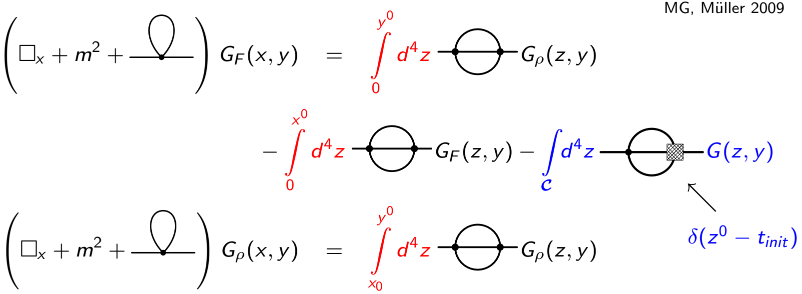
. . . encode the Non-Gaussian initial correlations

Initial 4-point correlation, 2PI three-loop truncation



Kadanoff-Baym equations for Non-Gaussian initial states

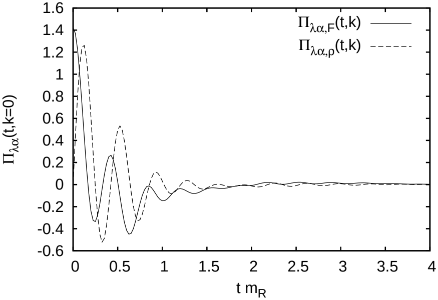
MG, Müller 2009



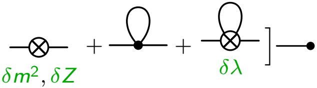
new Non-Gaussian contribution on the right-hand side:

$$\int_{\mathcal{C}} d^4z \, \Pi_{\lambda\alpha}(x,z) G(z,y)$$

$$= \int d^3z \, \Pi_{\lambda\alpha}(x,\mathbf{z}) G(z^0 = 0, \mathbf{z}, y)$$

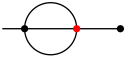
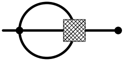


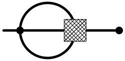
Correlation energy at initial time is non-zero !

$$E_{kin}(t=0) = \frac{1}{2}\Big[\partial_{x^0}\partial_{y^0} + \nabla^2 + m_R^2 +$$ 

$$\Big|_{x=0} + \delta\Lambda$$

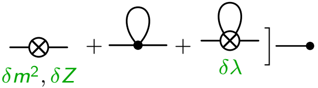
$$E_{corr}^{4-p.}(t=0) = \frac{i}{4}\int_{\mathcal{C}} d^4z\, [\Pi_{Gauss}(x,z) + \Pi_{non-Gauss}(x,z)]\, G(z,x)\Big|_{x=0}$$

$$= \underbrace{\int_0^t dz^0}_{\to 0}$$  $$\Big|_{x=0} +$$  $$\Big|_{x=0}$$

$$=$$  $$\Big|_{x=0}$$

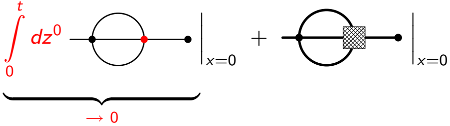
Correlation energy at initial time is non-zero !

$$E_{kin}(t=0) = \frac{1}{2}\Big[\partial_{x^0}\partial_{y^0} + \nabla^2 + m_R^2 + \ldots\Big] \ldots \Big|_{x=0} + \delta\Lambda$$

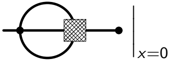


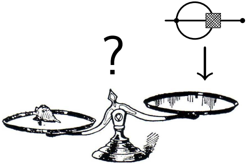
$$E_{corr}^{4-p.}(t=0) = \frac{i}{4}\int_{\mathcal{C}} d^4z\,[\Pi_{Gauss}(x,z) + \Pi_{non-Gauss}(x,z)]\,G(z,x)\Big|_{x=0}$$

$$= \int_0^t dz^0 \ldots \Big|_{x=0} + \ldots \Big|_{x=0}$$



$$= \ldots \Big|_{x=0}$$



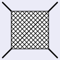
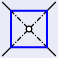


Physical initial state *MG, Müller (2009)*

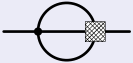
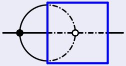
$$\alpha_n(x_1,\ldots,x_n) = \alpha_n^{th}(x_1,\ldots,x_n) + \Delta\alpha_n(x_1,\ldots,x_n)$$

Example: 2PI three-loop (setting-sun) approximation

$$\alpha_4^{th,2PI\ 3L}(z_1,z_2,z_3,z_4) \quad = \quad -i\lambda \int_{\mathcal{I}} d^4v\ \Delta(v,z_1)\Delta(v,z_2)\Delta(v,z_3)\Delta(v,z_4)$$

 = 

where $\Delta(v,z) \sim G_{th}(v)\delta(z^0)$

$\Pi_{\lambda\alpha}^{th,2PI\ 3L}(x,z)\big|_{\alpha_4}$ =  = 

Leading Non-Gaussian correction

Gaussian IC

$$
\begin{aligned}
G(x,y)|_{x^0,y^0=0} &= G_{th}(x,y)|_{x^0,y^0=0} \\
\alpha_4(x_1,\ldots,x_4) &= 0 \\
\alpha_n(x_1,\ldots,x_n) &= 0 \quad \text{for } n>4
\end{aligned}
$$

Non-Gaussian IC with α_4^{th}

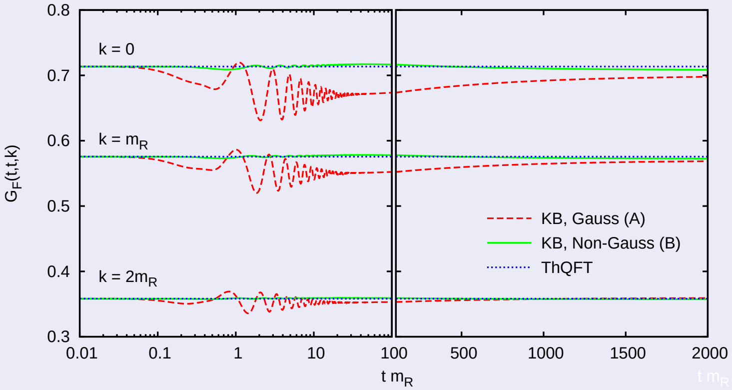
$$
\begin{aligned}
G(x,y)|_{x^0,y^0=0} &= G_{th}(x,y)|_{x^0,y^0=0} \\
\alpha_4(x_1,\ldots,x_4) &= \alpha_4^{th}(x_1,\ldots,x_4) \\
\alpha_n(x_1,\ldots,x_n) &= 0 \quad \text{for } n>4
\end{aligned}
$$

- Truncate thermal initial correlations
- $\Rightarrow$ *nonequilibrium* initial states
- The upper states are 'as thermal as possible'
- Expectation: Non-Gaussian state more close to equilibrium

Leading Non-Gaussian correction

Minimal offset from thermal equilibrium

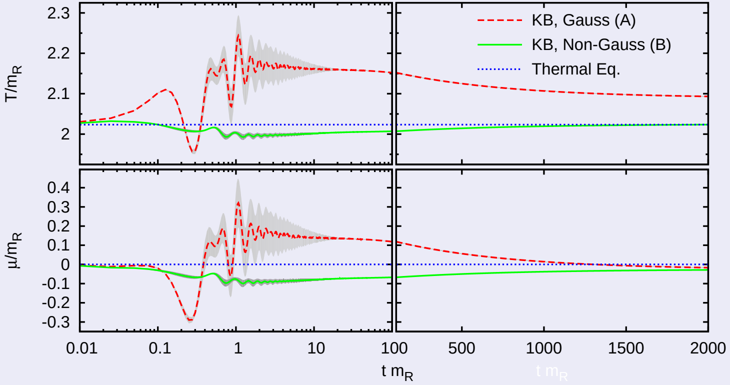
MG, Müller (2009)



Leading Non-Gaussian correction

Offset between initial and final temperature

MG, Müller (2009)



build-up   kinetic -   chemical equilibration

Leading Non-Gaussian correction

Gaussian IC

$$
\begin{aligned}
E_{total} &= E^{eq}_{kin}(T_{init}) \\
&= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final})
\end{aligned}
$$

$$E^{eq}_{corr} \sim \Lambda^4 + T^2\Lambda^2 + \ldots \Rightarrow$$

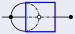
$$T_{init} \neq T_{final}$$

Cutoff-divergence

$$|T_{init} - T_{final}| \sim \Lambda^2$$

Non-Gaussian IC with α_4^{th}

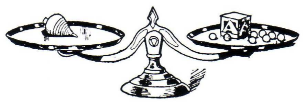
$$
\begin{aligned}
E_{total} &= E^{eq}_{kin}(T_{init}) + E^{eq}_{4-p.\,corr}(T_{init}) \\
&= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final})
\end{aligned}
$$

$E^{eq}_{4-p.\,corr} =$  $= E^{eq}_{corr} \Rightarrow$

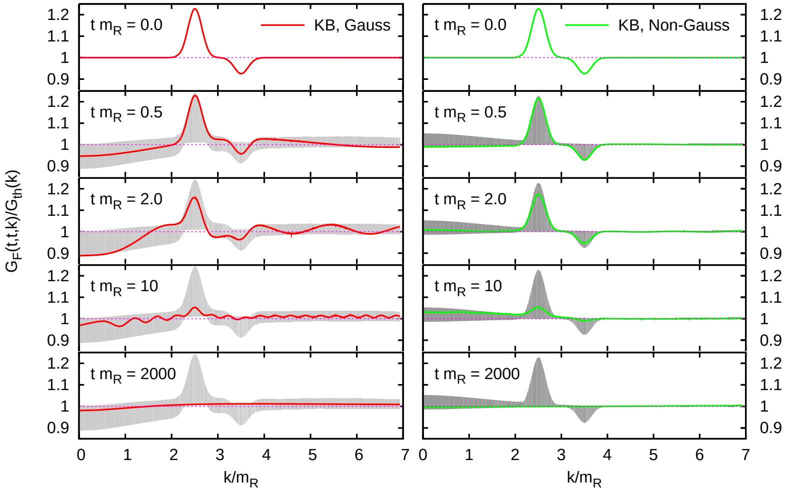
$$T_{init} = T_{final}$$

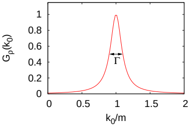
No Cutoff-divergence

$$E_{total} = E^{eq}(T_{init}) = E^{eq}(T_{final}) = \text{finite}$$



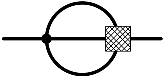
Renormalized Nonequilibrium Dynamics





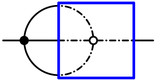
Kadanoff-Baym:

Quantum Boltzmann equations



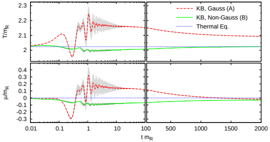
Renormalization:

Non-Gaussian initial state $\alpha_n(x_1, \ldots, x_n)$



Physical Initial State:

$$\alpha_n = {\color{blue}\alpha_n^{th}} + \Delta\alpha_n$$

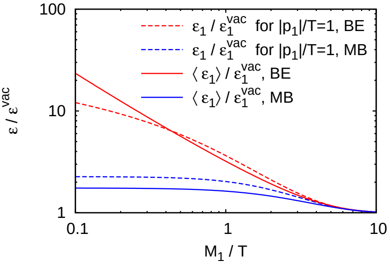


Leading Non-Gaussian Correction:

Removes Singularity

Outlook

Toy model of leptogenesis, top-down approach



MG, Lindner, Hohenegger, Kartavtsev

thank you!

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für Kernphysik
Heidelberg

MG, Markus Michael Müller, arXiv:0904.3600