Nonlocal Linear Cosmology

S. Yu. Vernov
Skobeltsyn Institute of Nuclear Physics,
Moscow State University, Moscow, Russia

based on
I.Ya. Aref'eva, L.V. Joukovskaya, S.V.,
J. Phys A **41** (2008) 304003, arXiv:0711.1364
A.S. Koshelev, S.V., arXiv:0903.5176
S.V., arXiv:0907.0468

To specify different types of cosmic fluids one uses a phenomenological relation between the pressure p and the energy density ϱ

$$p = w\varrho, \qquad p = E_k - V, \qquad \varrho = E_k + V$$

where w is the state parameter.

$$w(t) = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2E_k}{\varrho}. \tag{1}$$

Contemporary experiments give strong support that

$$w_{DE} = -1 \pm 0.2. \tag{2}$$

We consider the case $w_{DE} < -1$. Null energy condition (NEC) is violated and there are problems of instability. A possible way to evade the instability problem for models with $w_{DE} < -1$ is to yield a phantom model as an effective one, arising from a more fundamental theory.

In particular, if we consider some non-local models with operator

$$\phi e^{-\Box_g}\phi, \tag{3}$$

then we can get a kinetic term with a ghost sign.

Such a possibility appears in the string field theory framework:

I.Ya. Aref'eva, astro-ph/0410443, 2004.
I.Ya. Aref'eva and L.V. Joukovskaya, 2005;
I.Ya. Aref'eva and A.S. Koshelev, 2006; I.Ya. Aref'eva and A.S. Koshelev, 2008;
I.Ya. Aref'eva and I.V. Volovich, 2006; I.Ya. Aref'eva and I.V. Volovich, 2007;
I.Ya. Aref'eva, 2007; A.S. Koshelev, 2007;
L.V. Joukovskaya, 2007, L.V. Joukovskaya, 2008;
J.E. Lidsey, 2007;
G. Calcagni, 2006; G. Calcagni, M. Montobbio and G. Nardelli; 2007;
G. Calcagni and G. Nardelli; 2007; G. Calcagni and G. Nardelli; 2009;
N. Barnaby, T. Biswas and J.M. Cline, 2006; N. Barnaby and J.M. Cline, 2007; N. Barnaby and N. Kamran, 2007, 2008; N. Barnaby, 2008.
D.J. Mulryne and N.J. Nunes, 2008

Non-local action in the general form

Let us consider the following nonlocal action:

$$S = \int d^4x \sqrt{-g}\alpha' \left(\frac{R}{16\pi G_N} + \frac{1}{2g_o^2}\phi \mathcal{F}(\Box)\phi - \Lambda \right) \qquad (4)$$

Here G_N **is the Newton constant:** $8\pi G_N = 1/M_P^2$**.**

Function $\mathcal{F}$ **is assumed to be an analytic function**

$$\mathcal{F}(\Box_g) = \sum_{n=0}^{\infty} f_n \Box_g^n. \qquad (5)$$

From the SFT after some approximations we obtained:

$$\mathcal{F}_{SFT}(\Box_g) = (\xi^2 \Box_g + 1)e^{-2\Box_g} - c, \qquad (6)$$

where c **and** ξ^2 **are constants.**

$\mathcal{F}_{SFT}(\Box_g)$ **has only simple and (for some values of** c **and** ξ^2**) double roots.**

We consider the case of an arbitrary analytic $\mathcal{F}$**, which has only simple and double roots.**

In an arbitrary metric the energy-momentum tensor is

$$T_{\mu\nu} \ = \ \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \Big(\partial_\mu\Box^l\phi\partial_\nu\Box^{n-1-l}\phi + \partial_\nu\Box^l\phi\partial_\mu\Box^{n-1-l}\phi - \\ - g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\Box^l\phi\partial_\sigma\Box^{n-1-l}\phi + \Box^l\phi\Box^{n-l}\phi\right)\Big)$$

(A. Koshelev, 2007) and can be presented in the following form:

$$T_{\mu\nu} = E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu}\left(g^{\rho\sigma}E_{\rho\sigma} + V\right), \tag{7}$$

where

$$E_{\mu\nu} \equiv \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu\Box^l\phi\partial_\nu\Box^{n-1-l}\phi, \tag{8}$$

$$V \equiv \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \Box^l\phi\Box^{n-l}\phi. \tag{9}$$

Equations of motion are

$$G_{\mu\nu} = \frac{8\pi G_N}{g_o^2} T_{\mu\nu} + 8\pi G_N \Lambda, \tag{10}$$

$$\mathcal{F}(\Box)\phi = 0. \tag{11}$$

It is a system of nonlocal nonlinear equations!!!

Equations of motion are

$$G_{\mu\nu} = \frac{8\pi G_N}{g_o^2} T_{\mu\nu} + 8\pi G_N \Lambda, \quad (12)$$

$$\mathcal{F}(\Box)\phi = 0. \quad (13)$$

It is a system of nonlocal nonlinear equations!!!

HOW CAN WE FIND A SOLUTION ?

Equations of motion are

$$G_{\mu\nu} = \frac{8\pi G_N}{g_o^2} T_{\mu\nu} + 8\pi G_N \Lambda, \tag{14}$$

$$\mathcal{F}(\Box)\phi = 0. \tag{15}$$

It is a system of nonlocal nonlinear equations!!!

HOW CAN WE FIND A SOLUTION?

We assume that the metric $g_{\mu\nu}$ is given and consider (15) as an equation in ϕ.

The eigenfunctions of the Beltrami–Laplace operator

$$\Box_g \phi \equiv \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \phi = M\phi, \tag{16}$$

also represent the solutions of equation of motion (15) with M, which is defined as a solution of the characteristic equation

$$\mathcal{F}(M) = 0. \tag{17}$$

The characteristic equation does not depend on metric!

If $\mathcal{F}(M)$ has only simple roots M_k, then the solution of

$$\mathcal{F}(\Box)\phi = 0 \tag{18}$$

is

$$\phi = \sum_{k=1}^{N} \phi_k, \tag{19}$$

where $\Box\phi_k = M_k\phi_k$.

If $\mathcal{F}(M)$ has both single roots M_i and double roots $\tilde{M}_k$, then the solution is

$$\phi_0 = \sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k, \tag{20}$$

where

$$(\Box - M_i)\phi_i = 0, \qquad (\Box - \tilde{M}_k)^2\tilde{\phi}_k = 0. \tag{21}$$

The fourth order differential equation

$$(\Box - \tilde{M}_k)(\Box - \tilde{M}_k)\tilde{\phi}_k = 0 \tag{22}$$

is equivalent to the following system of equations:

$$(\Box - \tilde{M}_k)\tilde{\phi}_k = \varphi_k, \qquad (\Box - \tilde{M}_k)\varphi_k = 0. \tag{23}$$

Energy–momentum tensor for special solutions

If we have *one simple root* ϕ_1 such that $\Box\phi_1 = M_1\phi_1$, then

$$E_{\mu\nu}(\phi_1) = \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} M_1^{n-1}\partial_\mu\phi_1\partial_\nu\phi_1 = \frac{\mathcal{F}'(M_1)}{2}\partial_\mu\phi_1\partial_\nu\phi_1.$$

$$V(\phi_1) = \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} M_1^{n}\phi_1^2 = \frac{M_1}{2}\sum_{n=1}^{\infty} f_n n M_1^{n-1}\phi_1^2 = \frac{M_1\mathcal{F}'(M_1)}{2}\phi_1^2.$$

In the case of *two simple roots* ϕ_1 and ϕ_2 we have

$$E_{\mu\nu}(\phi_1+\phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2) + E_{\mu\nu}^{cr}(\phi_1,\phi_2), \tag{24}$$

where the cross term

$$E_{\mu\nu}^{cr}(\phi_1,\phi_2) = A_1\partial_\mu\phi_1\partial_\nu\phi_2 + A_2\partial_\mu\phi_2\partial_\nu\phi_1. \tag{25}$$

$$A_1 = \frac{\mathcal{F}(M_1)-\mathcal{F}(M_2)}{2(M_1-M_2)} = 0, \qquad A_2 = 0. \tag{26}$$

So, the cross term $E_{\mu\nu}^{cr}(\phi_1,\phi_2) = 0$ and

$$E_{\mu\nu}(\phi_1+\phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2) \tag{27}$$

Similar calculations shows

$$V(\phi_1 + \phi_2) = V(\phi_1) + V(\phi_2). \tag{28}$$

In the case of N *simple roots* the following formula has been obtained (A.S. Koshelev, S.V., 2009):

$$T_{\mu\nu} = \sum_{k=1}^{N} \mathcal{F}'(M_k) \left(\partial_\mu \phi_k \partial_\nu \phi_k - \frac{1}{2} g_{\mu\nu} \left(g^{\rho\sigma} \partial_\rho \phi_k \partial_\sigma \phi_k + M_k \phi_k^2 \right) \right). \tag{29}$$

Note that the last formula is exactly the energy-momentum tensor of many free massive scalar fields. If $\mathcal{F}(M)$ has simple real roots, then positive and negative values of $\mathcal{F}'(M_i)$ alternate, so we can obtain phantom fields.

Let $\tilde{M}_1$ is a double root. The fourth order differential equation $(\Box-\tilde{M}_1)^2\tilde{\phi}_1=0$ is equivalent to the following system of equations:

$$(\Box-\tilde{M}_1)\tilde{\phi}_1=\varphi_1, \qquad (\Box-\tilde{M}_1)\varphi_1=0. \tag{30}$$

It is convenient to write $\Box^l\tilde{\phi}_1$ in terms of the $\tilde{\phi}_1$ and φ_1:

$$\Box^l\tilde{\phi}_1=\tilde{M}_1^l\tilde{\phi}_1+l\tilde{M}_1^{l-1}\varphi_1. \tag{31}$$

Using (31) we obtain

$$E_{\mu\nu}(\tilde{\phi}_1)=B_1\partial_\mu\tilde{\phi}_1\partial_\nu\tilde{\phi}_1+B_2\partial_\mu\tilde{\phi}_1\partial_\nu\varphi_1+B_3\partial_\mu\phi_1\partial_\nu\tilde{\varphi}_1+B_4\partial_\mu\varphi_1\partial_\nu\varphi_1, \tag{32}$$

where

$$B_1=\frac{1}{2}\sum_{n=1}^{\infty}f_n n\tilde{M}_1^{n-1}=\frac{\mathcal{F}'(\tilde{M}_1)}{2}=0, \quad B_2=\frac{\mathcal{F}''(\tilde{M}_1)}{4}, \quad B_3=\frac{\mathcal{F}''(\tilde{M}_1)}{4}.$$

$$B_4=\frac{1}{12}\sum_{n=1}^{\infty}f_n n(n-1)(n-2)\tilde{M}_1^{n-3}=\frac{\mathcal{F}'''(\tilde{M}_1)}{12}.$$

Thus, for one double root we obtain the following result:

$$E_{\mu\nu}(\tilde{\phi}_1)=\frac{\mathcal{F}''(\tilde{M}_1)}{4}(\partial_\mu\tilde{\phi}_1\partial_\nu\varphi_1+\partial_\mu\phi_1\partial_\nu\tilde{\varphi}_1)+\frac{\mathcal{F}'''(\tilde{M}_1)}{12}\partial_\mu\varphi_1\partial_\nu\varphi_1.$$

Similar calculations gives

$$V(\tilde{\phi}_1) = \frac{\tilde{M}_1\mathcal{F}''(\tilde{M}_1)}{2}\tilde{\phi}_1\varphi_1 + \left(\frac{\tilde{M}_1\mathcal{F}'''(\tilde{M}_1)}{12} + \frac{\mathcal{F}''(\tilde{M}_1)}{4}\right)\varphi_1^2. \qquad (33)$$

For one single root and one double root we obtain:

$$E_{\mu\nu}(\tilde{\phi}_1 + \phi_2) = E_{\mu\nu}(\tilde{\phi}_1) + E_{\mu\nu}(\phi_2) + E^{cr}_{\mu\nu}(\tilde{\phi}_1, \phi_2), \qquad (34)$$

where

$$E^{cr}_{\mu\nu}(\tilde{\phi}_1, \phi_2) = B_5\partial_\mu\tilde{\phi}_1\partial_\nu\phi_2 + B_6\partial_\nu\tilde{\phi}_1\partial_\mu\phi_2 + B_7\partial_\mu\varphi_1\partial_\nu\phi_2 + B_8\partial_\nu\varphi_1\partial_\mu\phi_2.$$

It is easy to calculate:

$$E^{cr}_{\mu\nu}(\tilde{\phi}_1, \phi_2) = 0. \qquad (35)$$

Using similar calculations we obtain that

$$V(\tilde{\phi}_1 + \phi_2) = V(\tilde{\phi}_1) + V(\phi_2). \qquad (36)$$

Therefore, we obtain the following formula

$$T_{\mu\nu}\left(\tilde{\phi}_1 + \sum_{k=1}^{N}\phi_k\right) = T_{\mu\nu}\left(\tilde{\phi}_1\right) + T_{\mu\nu}\left(\sum_{k=1}^{N}\phi_k\right), \qquad (37)$$

where

$$T_{\mu\nu}(\tilde{\phi}_1) = E_{\mu\nu}(\tilde{\phi}_1) + E_{\nu\mu}(\tilde{\phi}_1) - g_{\mu\nu}\left(g^{\rho\sigma}E_{\rho\sigma}(\tilde{\phi}_1) - V(\tilde{\phi}_1)\right). \qquad (38)$$

So, we conclude that in the case of one double root the energy-momentum tensor can be separated into energy-momentum tensors for different modes of nonlocal scalar field, which corresponds to different roots of $\mathcal{F}$.

Let us consider the case of *two double roots* $\tilde{M}_1$ and $\tilde{M}_2$. We can write

$$E_{\mu\nu}(\tilde{\phi}_1 + \tilde{\phi}_2) = E_{\mu\nu}(\tilde{\phi}_1) + E_{\mu\nu}(\tilde{\phi}_2) + E^{cr}_{\mu\nu}(\tilde{\phi}_1, \tilde{\phi}_2), \qquad (39)$$

where

$$E^{cr}_{\mu\nu}(\tilde{\phi}_1, \tilde{\phi}_2) = 0. \qquad (40)$$

The results, obtained for two summands, can be straightforwardly generalize on *an arbitrary number of summands*.

The results, obtained for two summands, can be straightforwardly generalize on *an arbitrary number of summands*.

So, we obtain that for any analytical function $\mathcal{F}$, which has simple roots M_i and double roots $\tilde{M}_k$, the energy-momentum tensor

$$T_{\mu\nu}\left(\sum_{i=1}^{N_1}\phi_i+\sum_{k=1}^{N_2}\tilde{\phi}_k\right)=\sum_{i=1}^{N_1}T_{\mu\nu}(\phi_i)+\sum_{k=1}^{N_2}T_{\mu\nu}(\tilde{\phi}_k). \tag{41}$$

The result has been obtained for an arbitrary metric $g_{\mu\nu}$.

From now on the only metric we will be interested in is the spatially flat Friedmann–Robertson–Walker (FRW) metric of the form

$$ds^2 = -\,dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right) \qquad (42)$$

where $a(t)$ **is the scale factor,** $H = \dot{a}/a$ **and the dot denotes time derivative. A solution** ϕ **depends only on time.**

From now on the only metric we will be interested in is the spatially flat Friedmann–Robertson–Walker (FRW) metric of the form

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right) \tag{43}$$

where $a(t)$ **is the scale factor,** $H = \dot{a}/a$ **and the dot denotes time derivative. A solution** ϕ **depends only on time.**

The energy-momentum tensor in (14) in this metric can be written in the form of a perfect fluid $T^{\mu}_{\nu} = \mathrm{diag}(-\varrho, p, p, p)$**, where**

$$\begin{aligned}
\varrho &= \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{l=n-1}\left(\partial_t\Box^l\phi\partial_t\Box^{n-1-l}\phi + \Box^l\phi\Box^{n-l}\phi\right), \\
p &= \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{l=n-1}\left(\partial_t\Box^l\phi\partial_t\Box^{n-1-l}\phi - \Box^l\phi\Box^{n-l}\phi\right).
\end{aligned} \tag{44}$$

The Friedmann equations

$$\begin{cases} 3H^2 = 8\pi G_N \dfrac{\varrho}{g_o^2} + \Lambda, \\ \quad \dot{H} = \ - 4\pi G(\varrho + p). \end{cases} \tag{45}$$

The consequence of (45) is the conservation equation:

$$\dot{\varrho} + 3H(\varrho + p) = 0. \tag{46}$$

Note that system (45) is a non-local and non-linear system of equation.

For the field

$$\phi = \sum_{k=1}^{N} \phi_k, \tag{47}$$

where $\Box\phi_k = M_k\phi_k$, we obtain:

$$\varrho = \frac{1}{2}\sum_i \mathcal{F}'(M_i)\left(\dot{\phi}_i^2 + M_i\phi_i^2\right),$$

$$p = \frac{1}{2}\sum_i \mathcal{F}'(M_i)\left(\dot{\phi}_i^2 - M_i\phi_i^2\right).$$

Therefore, we can rewrite system (45) as follows:

$$\left\{\begin{aligned} 3H^2 &= 4\pi G \sum_i \mathcal{F}'(M_i)\left(\dot{\phi}_i^2 + M_i\phi_i^2\right) + 8\pi G_N \Lambda, \\ \dot{H} &= -4\pi G \sum_i \mathcal{F}'(M_i)\dot{\phi}_i^2. \end{aligned}\right. \qquad (48)$$

It is easy to check that equations for ϕ_i in (47) and (48) coincide with the Einstein equation for the following action:

$$S_{loc} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} + \frac{1}{2g_o^2}\sum_i \mathcal{F}'(M_i)\left(-g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i - M_i\phi_i^2\right) - \Lambda\right)$$

on the solutions with the FRW metric.

Here number of local scalar fields is equal to the number of roots of the characteristic equation. This system is equivalent to the initial non-local one because any solution to the equation of motion for the non-local field ϕ.

It may turn out that even if there infinitely many derivatives in the non-local system it can be equivalent in the above sense to the local one with finite number of fields. As an example one can take $\mathcal{F}(M) = (M+1)e^M$ with only one root $M = -1$.

For $\mathcal{F}(M)$ with two or more simple real roots we obtain that the non-local model with action (4) contains ghost-like excitations.

Note that the use of truncated function

$$\hat{\mathcal{F}}(\Box) \equiv \sum_{n=0}^{\hat{N}} f_n \Box^n \qquad (49)$$

instead of $\mathcal{F}(\Box)$ as an approximation is not correct, because $\hat{\mathcal{F}}$, which is the $\hat{N}$-th degree polynomial, contains spurious zeros, which are not present in $\mathcal{F}(\Box)$. The corresponding solution ϕ contains modes ϕ_i, which are not present in the full theory.

The detailed analysis of the initial value problem for such non-local equations and their mathematical properties can be found in (*N. Barnaby and N. Kamran, 2007*).

Double roots

For space-homogeneous solutions

$$\varrho = E_{00} + V \quad \textbf{and} \quad p = E_{00} - V. \tag{50}$$

From (45) we obtain the equation

$$-6HE_{00} = \dot{E}_{00} + \dot{V}. \tag{51}$$

In the case of one double root $\tilde{\phi}_1$ we substitute explicit expressions for

$$E_{00}(\tilde{\phi}_1) = \frac{\mathcal{F}''(\tilde{M}_1)}{2}\dot{\tilde{\phi}}_1\dot{\varphi}_1 + \frac{\mathcal{F}'''(\tilde{M}_1)}{12}\dot{\varphi}_1^2, \tag{52}$$

$$\dot{E}_{00}(\tilde{\phi}_1) = \frac{\mathcal{F}''(\tilde{M}_1)}{2}(\ddot{\tilde{\phi}}_1\dot{\varphi}_1 + \dot{\tilde{\phi}}_1\ddot{\varphi}_1) + \frac{\mathcal{F}'''(\tilde{M}_1)}{6}\ddot{\varphi}_1\dot{\varphi}_1, \tag{53}$$

$$\dot{V}(\tilde{\phi}) = \frac{\tilde{M}_1\mathcal{F}''(\tilde{M}_1)}{2}(\dot{\tilde{\phi}}_1\varphi_1 + \tilde{\phi}_1\dot{\varphi}_1) + \left(\frac{\tilde{M}_1\mathcal{F}'''(\tilde{M}_1)}{6} + \frac{\mathcal{F}''(\tilde{M}_1)}{2}\right)\varphi_1\dot{\varphi}_1. \tag{54}$$

Equation (51) can be written in the following form:

$$\frac{A_2}{2}\mathcal{F}''(\tilde{M}_1) + \frac{A_3}{6}\mathcal{F}'''(\tilde{M}_1) = 0, \tag{55}$$

where

$$A_2 = \dot{\tilde{\phi}}_1(\ddot{\varphi}_1 + 3H\dot{\varphi}_1 + \tilde{M}_1\varphi_1) + \dot{\varphi}_1(\ddot{\tilde{\phi}}_1 + 3H\dot{\tilde{\phi}}_1 + \tilde{M}_1\tilde{\phi} + \varphi_1), \tag{56}$$

$$A_3 = \dot{\varphi}_1(\ddot{\varphi}_1 + 3H\dot{\varphi}_1 + \tilde{M}_1\varphi_1). \tag{57}$$

We consider the case of an arbitrary function $\mathcal{F}(M)$, hence, we have no restrictions on $\mathcal{F}''(\tilde{M}_1)$ and $\mathcal{F}'''(\tilde{M}_1)$.

Coefficients A_2 and A_3 can be written via the box operator:

$$A_2 = \dot{\tilde{\phi}}_1(-\Box\varphi_1 + \tilde{M}_1\varphi_1) + \dot{\varphi}_1(-\Box\tilde{\phi}_1 + \tilde{M}_1\tilde{\phi} + \varphi_1), \tag{58}$$

$$A_3 = \dot{\varphi}_1(-\Box\varphi_1 + \tilde{M}_1\varphi_1). \tag{59}$$

We see that equation (51) is satisfied, because

$$\Box\varphi_1 = \tilde{M}_1\varphi_1, \tag{60}$$

$$\Box\tilde{\phi}_1 = \tilde{M}_1\tilde{\phi}_1 + \varphi_1. \tag{61}$$

In the case of many modes we have the similar result. Considering the following local action

$$S_{loc} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2g_o^2}\sum_{i=1}^{N_1}\mathcal{F}'(M_i)\left(g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i + M_i\phi_i^2\right) - \right.$$
$$-\sum_{k=1}^{N_2}\left(g^{\mu\nu}\left(\frac{\mathcal{F}''(\tilde{M}_k)}{2}\partial_\mu\tilde{\phi}_k\partial_\nu\varphi_k + \frac{\mathcal{F}'''(\tilde{M}_1)}{6}\partial_\mu\varphi_k\partial_\nu\varphi_k\right) + \right.$$
$$\left.\left. + \frac{\tilde{M}_k\mathcal{F}''(\tilde{M}_k)}{2}\tilde{\phi}_k\varphi_k + \left(\frac{\tilde{M}_k\mathcal{F}'''(\tilde{M}_k)}{12} + \frac{\mathcal{F}''(\tilde{M}_k)}{4}\right)\varphi_k^2\right) + \Lambda\right),$$

we can see that both the Friedmann equation and equations on ϕ_k, $\tilde{\phi}_k$ and φ_k can be obtained from this action. So, our way of localization is self-consistent in the case of double roots as well.

Conclusions

We have studied the SFT inspired nonlocal models with quadratic potentials and obtained:

1. Roots of the characteristic equation do not depend on metric.

2. In an arbitrary metric the energy-momentum tensor for an arbitrary N-mode solution is a sum of the energy-momentum tensors for the corresponding one-mode solutions. This result is obtained for any analytic function $\mathcal{F}$ with single and double roots.

3. Our linear model with one nonlocal scalar field generates an infinite number of local models. Hence, special solutions for nonlocal model can be obtained from local (differential) equations. We have constructed (arXiv:0711.1364) an exact kink-type solution.

Local and non-local Einstein equations have one and the same solutions.
Nonlocality arises in the case of $\mathcal{F}(\Box_g)$ with an infinite number of roots.
One system of non-local Einstein equations $\Leftrightarrow$ Infinity number of systems of local Einstein equations.
These local systems be solved independently and gives different particular solutions of the initial non-local system.

Local and non-local Einstein equations have one and the same solutions.
Nonlocality arises in the case of $\mathcal{F}(\Box_g)$ with an infinite number of roots.
One system of non-local Einstein equations $\Leftrightarrow$ Infinity number of systems of local Einstein equations.
These local systems be solved independently and gives different particular solutions of the initial non-local system.

I.Ya. Aref'eva, L.V. Joukovskaya, S.V., *J. Phys A* 41 (2008) 304003, arXiv:0711.1364;
A.S. Koshelev, S.V., arXiv:0903.5176;
S.V., arXiv:0907.0468.