Emergent Gravity from Matrix Models: Reconciling Cosmology with Quantum Mechanics ?

Harold Steinacker

Department of Physics, University of Vienna

PASCOS 2009

Motivation

- expect quantum structure of space-time at Planck scale

  due to Gravity $\leftrightarrow$ Quantum Mechanics

- QM & GR $\rightarrow$ cosmological constant problem: huge discrepancy

$$\frac{\Lambda_{\rm QM}}{\Lambda_{\rm concordance}} \geq 10^{60}$$

$\Rightarrow$ look for modified models of gravity on NC spaces

Matrix Models

$\leftrightarrow$ "noncommutative" (=quantized) spaces

Outline

- short intro to matrix models & emergent gravity
- cosmological solutions & implications

IKKT (type IIB) Matrix Model:

IKKT (1996)

$$S_{YM} = Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} + \overline{\Psi}\Gamma_a[X^a, \Psi]$$

$X^a \in L(\mathcal{H}), \qquad a = 1, ..., 10$ ... hermitian matrices

$D = 10$ required by Q.M. consistency (maximal SUSY)

main features

- known to describe NC gauge theory, gravity expected
- new: mechanism, description of general geometry
  emergent gravity $\leftrightarrow$ NC gauge thy
  Rivelles 2002, Yang 2006, H.S. 2007 ff, ...

  $\left\{ \begin{array}{l} \text{space-time} \\ \text{metric} \end{array} \right\}$ not fundamental, emerge

- expect good behavior under quantization
  may solve cosm.const. problem

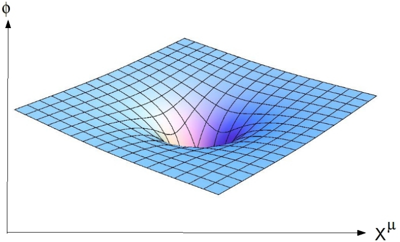
Geometry from Matrix Models

space-time as 3+1-dimensional NC brane solution: split matrices

$$X^a = (X^\mu, \Phi^i), \qquad \mu = 1, ..., 4;$$

$X^\mu \sim x^\mu$ ... (quantization of) coordinate function x^μ on $\mathcal{M}_\theta^4$

$\Phi^i \sim \Phi^i(x^\mu)$ ... embedding of 4D brane $\mathcal{M}_\theta^4 \subset \mathbb{R}^{10}$



e.o.m. $[X^a, [X^b, X^{a'}]]\eta_{aa'} = 0$,

solutions $[X^\mu, X^\nu] \sim i\theta^{\mu\nu}(x)$ ... quantized 4-D manifold $\mathcal{M}_\theta^4$ with Poisson structure

dynamical quantum (NC) space-time

Poisson structure and effective metric

- fluctuations of matrices around background
  $\Rightarrow$ scalar & gauge fields ($SU(n)$!), fermions on $\mathcal{M}^4$
- $[X^\mu, \phi(x)] \sim i\theta^{\mu\nu}\partial_\nu\phi(x)$

$\Rightarrow$ all fields couple to effective metric

$$\boxed{G^{\mu\nu}(x) = e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\; g_{\mu'\nu'}(x)}$$ (cf. open string metric)

where

$$e^{-\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|},$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \partial_\mu\Phi^i\partial_\nu\Phi^j\delta_{ij}$$ induced metric on $\mathcal{M}^4_\theta$

(cf. closed string metric)

all fields couple to effective metric $G_{\mu\nu}(x)$ (gauge, scalar, fermions)
general 4D geometry $\Longrightarrow$ gravity on $\mathcal{M}^4_\theta$

$su(n)$ gauge fields: same model, new vacuum

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$\boxed{S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu} F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F}$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

covariant equations of motion (semi-classical limit)

matrix e.o.m: $[X^a,[X^b,X^{a'}]]\eta_{aa'} = 0 \quad \Longleftrightarrow$ (H.S., NPB 810 (2009))

$$
\begin{aligned}
\Delta_G \Phi^i &= 0, \\
\nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta \\
\eta &= e^\sigma\, G^{\mu\nu} g_{\mu\nu}
\end{aligned}
$$

furthermore:

$\Delta_G x^\mu = 0$ (interpreted as scalar field)

$\Rightarrow$

$\mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ is harmonic embedding (w.r.t. $G_{\mu\nu}$)
minimal surface

Quantization and E-H action

Quantization of matrix model:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]}$$

note:

- bosonic M. M.: "measure" for integral over geometry
- expect: well-defined for IKKT model $(D = 10)$
- one-loop: fields couple to $G_{\mu\nu}$

  $\Rightarrow$ induced Einstein-Hilbert action:

$$S_{1-loop} \sim \int d^4x \sqrt{|G_{\mu\nu}|} \, (c_1 \Lambda_1^4 + c_2 \Lambda_4^2 \, R[G] + O(\log(\Lambda_{UV})))$$

Vacuum energy & minimal surfaces

different physical implication of $\int d^4x \sqrt{|G_{\mu\nu}|}\,\Lambda_1^4$:
note:

- $G_{\mu\nu}$ determined by embedding $\phi^i(x)$ & $\theta^{\mu\nu}$
- $|G_{\mu\nu}| \equiv |g_{\mu\nu}|$, independent of $\theta^{\mu\nu}$

$$\Rightarrow \quad \delta \int d^4x \sqrt{G} \sim \int d^4x \sqrt{g} g^{\mu\nu} \delta g_{\mu\nu} \sim \int d^4x \sqrt{g} \delta\phi^i \Delta_g \phi^j \delta_{ij}$$

vanishes for harmonic embeddings

$$\Delta_g \phi^i = 0$$

new solutions (besides GR): minimal surfaces,
stabilized by (huge) vacuum energy

$\int d^4x \sqrt{|G_{\mu\nu}|}\,\Lambda_1^4$ ... brane tension, not cosmological constant

harmonically embedded (e.g. flat) spaces stable,
protected from cosm. const. problem

Cosmological solution

... joint work with D. Klammer   D. Klammer, H. S., arXiv:0903.0986, PRL ...

<u>Assume:</u> vacuum energy $\Lambda^4 \gg$ energy density ρ

$\Rightarrow$ look for harmonic embedding $\Delta x^a = 0$ of FRW metric

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + \sinh^2(\chi) d\Omega^2),$$

Ansatz

$$x^a(t,\chi,\theta,\varphi) = \begin{pmatrix} a(t) \begin{pmatrix} \cos\psi(t) \\ \sin\psi(t) \end{pmatrix} \otimes \begin{pmatrix} \sinh(\chi)\sin\theta\cos\varphi \\ \sinh(\chi)\sin\theta\sin\varphi \\ \sinh(\chi)\cos\theta \\ \cosh(\chi) \end{pmatrix} \\ 0 \\ x_c(t) \end{pmatrix} \in \mathbb{R}^{10}$$

Evolution $a(t), \Psi(t), x_c(t)$ determined by $\Delta x^a = 0$

solution of M.M + leading term $\int d^4x \sqrt{G}\Lambda^4$ in Γ_{1-loop}

$\theta^{\mu\nu}$ determined by $\nabla^\mu \theta^{-1}_{\mu\nu} = 0, \quad \star(\theta^{-1}) = \pm i\theta^{-1} \; (\Rightarrow g_{\mu\nu} = G_{\mu\nu})$

Cosmological solution

$\Delta_g x_c = 0 = \Delta_g(\mathcal{R}(t)S(\chi)\cos\theta)$ leads to

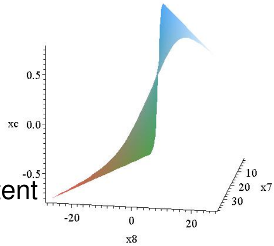
$$\begin{aligned} 3\tfrac{1}{a}(\dot{a}^2+k)+\ddot{a}-\dot{\psi}^2 a &= 0 \\ 5\dot{\psi}\dot{a}+\ddot{\psi}a &= 0 \\ 3\tfrac{1}{a}\dot{a}\dot{x}_c+\ddot{x}_c &= 0. \end{aligned}$$

integrated:

$$\begin{aligned} (\dot{a}^2+k)a^6+b^2a^{-2} &= d^2 = const \\ \dot{\psi} &= b\,a^{-5}, \qquad b = const > 0 \\ \dot{x}_c &= d\,a^{-3}; \end{aligned}$$

analog of Friedmann equations

$$\begin{aligned} H^2 = \frac{\dot{a}^2}{a^2} &= -b^2a^{-10}+d^2a^{-8}-\frac{k}{a^2}. \\ \frac{\ddot{a}}{a} &= -3d^2a^{-8}+4b^2a^{-10}. \end{aligned}$$



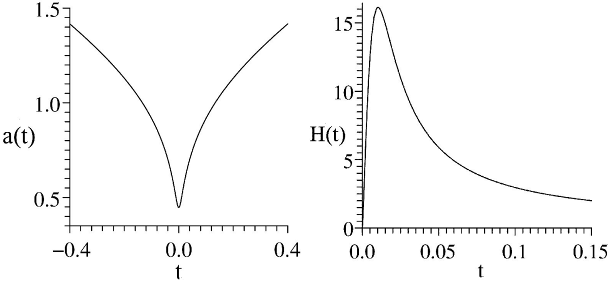
largely independent of detailed matter/energy content
as long as $\Lambda^4 \gg \rho$

$k = -1$ (negative spatial curvature) most interesting

Implications:

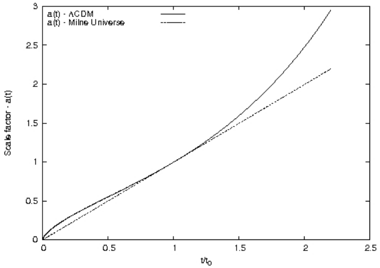
1) early universe:

- big bounce: $\dot{a} = 0$ for $a = a_{min} \sim b^{1/4}$
  ($\exists$ bound for energy density ρ vs. vacuum energy Λ^4)
- inflation-like phase $a(t) \sim t^2$, ends at $a(t_{\rm exit}) = \sqrt{\frac{4}{3}\frac{b}{d}}$
  geometric mechanism (no scalar field required),
  no fine-tuning



2) late evolution (now): $\dot{a} \to 1$

approaches Milne-like universe ($k = -1$, spatial curvature),



in remarkably good agreement with observation
(age $13.8 \cdot 10^9$ *yr*, type Ia supernovae)
different physics for early universe (recombination etc.)

A. Benoit-Levy and G. Chardin, [arXiv:0903.2446]

CMB acoustic peak argued to be at correct scale (?)

no fine-tuning of cosm. const., no need for dark energy !

further implications:

- gauge couplings will be different in early universe
- non-standard coupling of spin to gravity

Summary and Conclusion

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$

  brane solutions $\to$

  dynamical NC spaces $\leftrightarrow$ emergent gravity
- *not* same as G.R., E-H action induced
- intriguing cosmological solution,

  physics of vacuum energy different from GR

  no fine-tuning of cosm. const.
- suitable for quantizing gravity

  (IKKT model, $N = 4$ SUSY in $D = 4$)
- open problem: Schwarzschild, etc ... ??

Solution of $\theta^{-1}_{\mu\nu}$:

- FRW metric $ds^2_{FRW} = \alpha(t)(-d\tau^2 + dy_1^2 + dy_2^2 + dy_3^2)$ conf. flat
  solution of $\nabla^\mu \theta^{-1}_{\mu\nu} = 0$:

$$\theta^{-1} = i d\tau \wedge dy_1 + dy_2 \wedge dy_3$$

(complexified $\leftrightarrow$ Wick rotation $x^0 = it$)

- SSB of Lorentz symmetry $\Rightarrow$ gravitational waves: recall

$$\nabla^\mu \theta^{-1}_{\mu\nu} = 0 \qquad (\text{for } g_{\mu\nu} = G_{\mu\nu})$$

... 2 propagating d.o.f. $\theta^{-1}_{\mu\nu} = \bar\theta^{-1}_{\mu\nu} + F_{\mu\nu}$ around flat $\mathbb{R}^4_\theta$
are actually gravitons

$$h_{\mu\nu} = -\bar G_{\nu\nu'}\bar\theta^{\nu'\rho}F_{\rho\mu} - \bar G_{\mu\mu'}\bar\theta^{\mu'\rho}F_{\rho\nu} - \frac{1}{2}\bar G_{\mu\nu}F_{\rho\eta}\bar\theta^{\rho\eta}$$

e.o.m. imply Rivelles hep-th/0212262

$$R_{\mu\nu}[\bar G + h] = 0 \quad + O(\theta^2), \qquad \text{while} \quad R_{\mu\nu\rho\sigma} \neq 0\,.$$

Solution of $\theta^{-1}_{\mu\nu}$:

- FRW metric $ds^2_{FRW} = \alpha(t)(-d\tau^2 + dy_1^2 + dy_2^2 + dy_3^2)$ conf. flat
  solution of $\nabla^\mu\theta^{-1}_{\mu\nu} = 0$:

$$\theta^{-1} = i d\tau \wedge dy_1 + dy_2 \wedge dy_3$$

(complexified $\leftrightarrow$ Wick rotation $x^0 = it$)

- SSB of Lorentz symmetry $\Rightarrow$ gravitational waves: recall

$$\nabla^\mu\theta^{-1}_{\mu\nu} = 0 \qquad (\text{for } g_{\mu\nu} = G_{\mu\nu})$$

... 2 propagating d.o.f. $\theta^{-1}_{\mu\nu} = \bar\theta^{-1}_{\mu\nu} + F_{\mu\nu}$ around flat $\mathbb{R}^4_\theta$
are actually gravitons

$$h_{\mu\nu} = -\bar G_{\nu\nu'}\bar\theta^{\nu'\rho}F_{\rho\mu} - \bar G_{\mu\mu'}\bar\theta^{\mu'\rho}F_{\rho\nu} - \frac{1}{2}\bar G_{\mu\nu}F_{\rho\eta}\bar\theta^{\rho\eta}$$

e.o.m. imply Rivelles hep-th/0212262

$$R_{\mu\nu}[\bar G + h] = 0 \quad + O(\theta^2), \qquad \text{while} \quad R_{\mu\nu\rho\sigma} \neq 0\,.$$