

Effective equations of motion on the brane in higher order dilaton gravity

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Outline

introduction

higher order dilaton gravity

effective brane equations

cosmological example

higher order & conclusions

why not to be perfectly happy with General Relativity?

- ▶ successful classical description of gravitational interactions
- ▶ overcame numerous consistency and experimental tests

- ▶ however, not without shortcomings
 - ↪ making the long story short
 - ▶ resistance to quantization
 - ▶ no unification of gauge interactions and gravity
 - ↪ no connection whatsoever to the Standard Model

- ▶ an extended theory of gravity seems to be necessary
 - with General Relativity as the main part of its structure

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what if 'some' extra dimensions existed ...

- ▶ currently observed 4 space-time dimensions
 - ▶ Einstein equations of motion
 - ▶ Einstein-Hilbert action
 - linear in Riemann tensor

- ▶ higher-dimensional space-times
 - ↪ additional higher curvature terms can be considered

- ▶ introducing higher powers of Riemann tensor into the gravity action
 - ↪ Einstein theory of gravity generalized

- ▶ for a given order in the Riemann tensor
 - contribution to the action unique (overall normalization)
 - ▶ quadratic contribution: Gauss-Bonnet (Lanczos) term
 - ▶ generalized to higher orders by Lovelock

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string theory behind the scenes?

- ▶ effective action obtained from string theories
 - ↪ higher derivative corrections to the gravity interactions
- ▶ first correction exactly of the form of the Gauss-Bonnet term
 - \pm local field redefinitions
- ▶ *dilaton*: scalar field of the gravitational sector
- ▶ α' expansion in the string theories
 - ↪ higher order corrections also for the dilaton interactions

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construction of the dilaton gravity equations of motion

- ▶ Einstein-Lovelock higher order gravity

↪ couple to the dilaton

- ▶ N -th order contributions to dilaton gravity equations of motion:

$$T_{\mu\nu}^{(N)} = 0 \ \& \ W^{(N)} = 0$$

↪ *constructed*

- ▶ at each order: unique up to a normalization

- ▶ *E-L theory generalized* → higher order dilaton gravity

- ▶ consistent treatment: *never done before*

some useful notation

- ▶ a generalization of the Kronecker delta

$$\delta_{\rho_1 \rho_2 \dots \rho_N}^{\sigma_1 \sigma_2 \dots \sigma_N} = \det \begin{vmatrix} \delta_{\rho_1}^{\sigma_1} & \delta_{\rho_2}^{\sigma_1} & \dots & \delta_{\rho_N}^{\sigma_1} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \delta_{\rho_1}^{\sigma_N} & \delta_{\rho_2}^{\sigma_N} & \dots & \delta_{\rho_N}^{\sigma_N} \end{vmatrix}$$

- ▶ a generalization of the trace operator

$$\mathcal{T}(M) = \delta_{\rho_1 \rho_2 \dots \rho_N}^{\sigma_1 \sigma_2 \dots \sigma_N} M^{\rho_1 \rho_2 \dots \rho_N}{}_{\sigma_1 \sigma_2 \dots \sigma_N}$$

- ▶ an extension of the trace operator

$$\bar{\mathcal{T}}_{\mu}^{\nu}(M) = \delta_{\mu \rho_1 \rho_2 \dots \rho_N}^{\nu \sigma_1 \sigma_2 \dots \sigma_N} M^{\rho_1 \rho_2 \dots \rho_N}{}_{\sigma_1 \sigma_2 \dots \sigma_N}$$

some useful notation → example

- ▶ and a generalization of the N -th power operator

$$\begin{aligned}
 & \mathcal{T} \left(\left[\frac{1}{2} \mathcal{R}_{**}^{**} \oplus 2(\nabla\nabla)_*^* \phi \right]^2 \right) = \\
 &= \frac{1}{4} \mathcal{T}(\mathcal{R}_{**}^{**} \mathcal{R}_{**}^{**}) + 2 \mathcal{T}(\mathcal{R}_{**}^{**} (\nabla\nabla)_*^* \phi) + 4 \mathcal{T}((\nabla\nabla)_*^* \phi (\nabla\nabla)_*^* \phi) = \\
 &= \frac{1}{4} \delta_{\rho_1 \rho_2 \rho_3 \rho_4}^{\sigma_1 \sigma_2 \sigma_3 \sigma_4} \mathcal{R}^{\rho_1 \rho_2}{}_{\sigma_1 \sigma_2} \mathcal{R}^{\rho_3 \rho_4}{}_{\sigma_3 \sigma_4} \\
 &\quad + 2 \delta_{\rho_1 \rho_2 \rho_3}^{\sigma_1 \sigma_2 \sigma_3} \mathcal{R}^{\rho_1 \rho_2}{}_{\sigma_1 \sigma_2} (\nabla^{\rho_3} \partial_{\sigma_3} \phi) \\
 &\quad + 4 \delta_{\rho_1 \rho_2}^{\sigma_1 \sigma_2} (\nabla^{\rho_1} \partial_{\sigma_1} \phi) (\nabla^{\rho_2} \partial_{\sigma_2} \phi)
 \end{aligned}$$

- ▶ asterisks → tensors ranks
- ▶ shorthand: $(\nabla\nabla)_\sigma^\rho \equiv \nabla^\rho \partial_\sigma$

(just acquired) starting point: d -dimensional higher order dilaton gravity

- ▶ d -dimensional tensor $T_{\mu\nu} = 0$ and scalar $W = 0$ equations of motion

$$-\sum_{N=1}^{N_{max}} \frac{\alpha_N}{2} \bar{T}_{\mu\nu} \left(\left[\frac{1}{2} \mathcal{R}_{**}^{**} \oplus 2(\nabla\nabla)_*^* \phi \oplus (-1)(\partial\phi)^2 \right]^N \right) + g_{\mu\nu} V(\phi) - \tau_{\mu\nu} \delta_B = 0$$

$$-\sum_{N=1}^{N_{max}} \frac{\alpha_N}{2} \mathcal{T} \left(\left[\frac{1}{2} \mathcal{R}_{**}^{**} \oplus 2(\nabla\nabla)_*^* \phi \oplus (-1)(\partial\phi)^2 \right]^N \right) + V(\phi) - V'(\phi) - \tau_\phi \delta_B = 0$$

- ▶ position of the brane: Dirac delta type distribution δ_B
- ▶ brane localized terms: $\tau_{\mu\nu} = h_{\mu\nu} \mathcal{L}_B - 2 \frac{\delta \mathcal{L}_B}{\delta h^{\mu\nu}}$ & $\tau_\phi = \mathcal{L}_B - \frac{\delta \mathcal{L}_B}{\delta \phi}$
due to the brane interactions given by \mathcal{L}_B
- ▶ induced brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$
 - ▶ n^μ : unit vector field normal to the brane (at the brane)
- ▶ corresponding Lagrangian density

$$\mathcal{L} = e^{-\phi} \left\{ \sum_{N=1}^{N_{max}} \frac{\alpha_N}{2} \mathcal{T} \left(\left[\frac{1}{2} \mathcal{R}_{**}^{**} \oplus 2(\nabla\nabla)_*^* \phi \oplus (-1)(\partial\phi)^2 \right]^N \right) - V(\phi) + \mathcal{L}_B \delta_B \right\}$$

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where to?

- ▶ **brane-world** ideology: Standard Model localized on a brane
 - embedded in higher dimensional space-time
 - ↪ how will the induced gravity look like on the brane?
- ▶ **effective equations of motion**: $(d - 1)$ -dimensional
 - ↪ simply restricting full d -dimensional equations? **NO!**
- ▶ certain quantities contributing to $T_{\mu\nu} = 0$ and $W = 0 \dots$ can be singular or discontinuous on the brane
 - ▶ singular: explicit Dirac delta contributions
or discontinuous functions derivatives
 - ↪ *non-trivial derivation* of effective equations on the brane
 - ↪ will be carried out in **COVARIANT APPROACH**

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projecting: “parallel” & “perpendicular” to the brane

- identification in relevant tensors: benefits from \mathcal{T} & $\overline{\mathcal{T}}$ antisymmetrization

$$\mathcal{R}_{**}^{**} \rightarrow R_{**}^{**} - 2K_*^* K_*^* - 4(nn)_*^* \{ \mathcal{L}_n K_*^* - (KK)_*^* \} - 8(nD)_*^* K_*^*$$

$$\begin{aligned} (\nabla\nabla)_*^* \phi &\rightarrow [(DD)_*^* \phi + K_*^* \mathcal{L}_n \phi] + (nn)_*^* \{ \mathcal{L}_n^2 \phi - a^e \nabla_e \phi \} \\ &\quad + 2[(nD)_*^* \mathcal{L}_n \phi - (nKD)_*^* \phi] \end{aligned}$$

$$(\partial\phi)^2 = (D\phi)^2 + (\mathcal{L}_n \phi)^2$$

- $g_{\mu\nu}$: $\mathcal{R}_{\rho\sigma}^{\mu\nu}$ & ∇_μ vs $h_{\mu\nu}$: $R_{\rho\sigma}^{\mu\nu}$ & D_μ
- $K_{\mu\nu}$: extrinsic curvature of hypersurfaces orthogonal to n^μ
- \mathcal{L}_n : Lie derivative along n^μ
- $a^e \nabla_e \phi = n^a (\nabla_a n^b) (\nabla_b \phi)$ (‘non-typical’; not present in final results)
- shorthand again: $(nn)_*^* \equiv n_* n^*$, $(DD)_*^* \equiv D_* D^*$, $(KK)_*^* \equiv K_X^* K_X^*$,
 $(nD)_*^* \equiv \frac{1}{2} (n_* D^* + n^* D_*)$, $(nKD)_*^* \equiv \frac{1}{2} (n_* K_X^* D^X + n^* K_X^* D_X)$

effective gravitational equations on the brane . . . identifying problems

- ▶ the good

→ $h_{\mu\nu}, R_{\mu\nu}, (DD)_{\mu\nu}\phi, (D\phi)^2, V(\phi)$

↪ no work needed here, rejoice!

- ▶ the kind of bad

→ $K_{\mu\nu}, \mathcal{L}_n\phi$

↪ can be discontinuous when 'crossing' the brane

- ▶ and the slightly ugly

→ $\mathcal{L}_n K_{\mu\nu}, \mathcal{L}_n^2\phi$

↪ can be singular on the brane

↪ can have a finite contribution as well

↪ all this information has to be properly taken into account

& the quest begins

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effective gravitational equations on the brane . . . addressing problems

- ▶ terms **discontinuous** on the brane (leading to singularities)

$$\rightarrow K_{\mu\nu}, \mathcal{L}_n \phi$$

\rightsquigarrow **junction** (boundary) **conditions**

- ▶ terms **singular** on the brane

$$\rightarrow \mathcal{L}_n K_{\mu\nu}, \mathcal{L}_n^2 \phi$$

- ▶ purely singular contributions already addressed by the junction conditions
- ▶ the smooth part has to be determined as well
(yields a finite contribution to the effective equations)

\rightsquigarrow **“brane limit of bulk equations system”**

- ▶ take scalar equation of motion & the trace of tensor equation of motion
 \rightarrow “bulk equations system”
- ▶ now the brane limit (i.e. evaluate ‘next to the brane’)
- ▶ solve it

effective gravitational equations on the brane . . . addressing problems

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effective gravitational equations on the brane . . .

- ▶ any order? complexity rather overwhelming
 - ↪ $N = 1$ & $d = 5$ example
 - ▶ *first order dilaton gravity*
 - ▶ and a 4-dimensional brane
- ▶ higher order dilaton gravity: exactly the same procedure
 - ↪ appropriate formulae derived just as well

effective gravitational equations ... bulk equations projected ($N = 1$)

- ▶ “parallel-parallel” part $T_{\mu\nu}^{\parallel\parallel} = 0$

$$\left[\left\{ R_{\mu\nu} + (DD)_{\mu\nu}\phi - \frac{h_{\mu\nu}}{2} \left(R + 2(DD)\phi - (D\phi)^2 \right) + h_{\mu\nu} \frac{V(\phi)}{\alpha_1} \right\} \right. \\ \left. + \left\{ ((KK)_{\mu\nu} - K_{\mu\nu}(K - \mathcal{L}_n\phi)) - \frac{h_{\mu\nu}}{2} \left((KK) - (K - \mathcal{L}_n\phi)^2 \right) \right\} \right. \\ \left. - \left\{ (\mathcal{L}_n K_{\mu\nu} - (KK)_{\mu\nu}) - h_{\mu\nu} \left((h\mathcal{L}_n K) - (KK) \right) - h_{\mu\nu} \left(\mathcal{L}_n^2 \phi - a^e \nabla_e \phi \right) \right\} \right]_{\pm} = 0$$

addressing the discontinuities' problem: junction conditions

- ▶ junction conditions for a given point x_0^μ on the brane:
 - integrating the d -dimensional equations of motion 'across-the-brane'
 - i.e. in the direction perpendicular to the brane
 - and shrinking the interval: 'infinitesimal across-the-brane integration'
- ▶ only some terms in the equations of motions \nrightarrow zero
 - ▶ explicit brane contributions proportional to δ_B
 - ▶ terms containing second Lie derivatives: $\mathcal{L}_n^2 \phi$ and $\mathcal{L}_n K_{\mu\nu}$
- ▶ useful notation
 - ▶ discontinuous at the brane \rightsquigarrow jump: $[f(x_0)]_\pm = [f(x_0)]_+ - [f(x_0)]_-$
 - ▶ "brane limits": $[f(x_0)]_+, [f(x_0)]_-$

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junction conditions: $N = 1 \rightsquigarrow [K_{\mu\nu}]_{\pm}$ & $[\mathcal{L}_n\phi]_{\pm}$ explicitly

- ▶ tensor junction condition (effectively from $T_{\mu\nu}^{\parallel\parallel} = 0$)
- ▶ scalar junction condition

\rightsquigarrow easily solvable, jumps can be determined

$$[K_{\mu\nu}]_{\pm} = \frac{1}{\alpha_1} (h_{\mu\nu}\tau_{\phi} - \tau_{\mu\nu})$$

$$[\mathcal{L}_n\phi]_{\pm} = \frac{1}{\alpha_1} ((d-2)\tau_{\phi} - \tau)$$

- ▶ however, no information whatsoever about the brane limits
 \rightsquigarrow unless ...

junction conditions: $N = 1$ & $\mathbb{Z}_2 \rightsquigarrow [K_{\mu\nu}]_+$ & $[\mathcal{L}_n\phi]_+$ explicitly

- ▶ \mathbb{Z}_2 symmetry, brane located at the orbifold fixed point

$$\rightsquigarrow [f]_{\pm} = 2[f]_+ \text{ if } f \text{ is } \mathbb{Z}_2\text{-odd, i.e. } [f]_- = -[f]_+$$

\rightsquigarrow brane limits can be determined

$$[K_{\mu\nu}]_+ = \frac{1}{2\alpha_1} (h_{\mu\nu}\tau_\phi - \tau_{\mu\nu})$$

$$[\mathcal{L}_n\phi]_+ = \frac{1}{2\alpha_1} ((d-2)\tau_\phi - \tau)$$

addressing the discontinuities' problem: bulk equations system

- ▶ terms singular on the brane
 - tend to appear in equations of motion in certain combinations
 - $\{\mathcal{L}_n K_{\mu\nu} - (KK)_{\mu\nu}\}, \{\mathcal{L}_n^2 \phi - a^e \nabla_e \phi\}$
 - ↪ good occasion to get rid of $a^e \nabla_e \phi$ as well
- ▶ “brane limit of bulk equations system”
 - ▶ supposed to yield finite contributions to $\mathcal{L}_n K_{\mu\nu}$ & $\mathcal{L}_n^2 \phi$
 - ▶ a system of scalar equations ($W = 0$ and trace of $T_{\mu\nu} = 0$)
 - ▶ but $\mathcal{L}_n K_{\mu\nu}$ is a tensor variable... how come it can work?
 - yes, it can

$$\begin{aligned} \{\mathcal{L}_n K_{\mu\nu} - (KK)_{\mu\nu}\} &= \frac{h_{\mu\nu}}{d-1} \left\{ (h \mathcal{L}_n K) - (KK) \right\} - \frac{1}{d-3} (R_{\mu\nu} - KK_{\mu\nu} + (KK)_{\mu\nu}) \\ &+ \frac{h_{\mu\nu}}{(d-1)(d-3)} (R - K^2 + (KK)) - \frac{d-2}{d-3} E_{\mu\nu} \end{aligned}$$

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addressing the discontinuities' problem: bulk equations system

- ▶ $E_{\mu\nu} \equiv C_{abcd} h_\mu^a n^b h_\nu^c n^d$, where C_{abcd} : bulk Weyl tensor
 - ↪ $E_{\mu\nu}$ enters $T_{\mu\nu}^{\parallel\parallel} = 0$
 - (so promising as effective gravitational equations on the brane)
 - to *never leave* it

- ▶ treating $T_{\mu\nu}^{\parallel\parallel} = 0$ as effective gravitational equations on the brane ...
 - ▶ single bulk associated variable $E_{\mu\nu}$
 - ↪ describes the permanent influence of bulk theory on brane-world gravity
 - ▶ *not a closed system*
 - ↪ bulk solutions essential to fully describe the gravity induced on the brane

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bulk equations system: $N = 1$

- ▶ two linear equations with two variables

$$g^{\mu\nu} T_{\mu\nu} = 0 \quad \& \quad W = 0$$

- ▶ non-zero determinant

$\rightsquigarrow \{h\mathcal{L}_n K\} - (KK)$ and $\{\mathcal{L}_n^2 \phi - a^e \nabla_e \phi\}$ can be determined uniquely

- ▶ $\{\mathcal{L}_n K_{\mu\nu} - (KK)_{\mu\nu}\}$ to be calculated subsequently

effective gravitational equations on the brane: $N = 1, d = 5, \mathbb{Z}_2$

- ▶ 4-dimensional brane

embedded in a 5-dimensional space-time with \mathbb{Z}_2 symmetry

↪ how shall we do it?

- ▶ take $T_{\mu\nu}^{\parallel\parallel} = 0$

- ▶ enter the solution of the brane limit of bulk equations system

↪ $\{(h\mathcal{L}_n K) - (KK)\}$ & $\{\mathcal{L}_n^2 \phi - a^e \nabla_e \phi\}$ & $\{\mathcal{L}_n K_{\mu\nu} - (KK)_{\mu\nu}\}$

- ▶ slightly readjust the relative $R_{\mu\nu}$ vs. R coefficient with $T^{\perp\perp} = 0$

↪ 'Einstein-like' form with the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R h_{\mu\nu}$

- ▶ enter the result of junction conditions analysis

↪ $[K_{\mu\nu}]_+$ & $[\mathcal{L}_n \phi]_+$

effective gravitational equations on the brane: $N = 1$, $d = 5$, \mathbb{Z}_2

↪ effective gravitational equations on the brane read

$$G_{\mu\nu} + E_{\mu\nu} + \frac{2}{3} \left((DD)_{\mu\nu} \phi - h_{\mu\nu} (DD) \phi \right) + \frac{1}{4} h_{\mu\nu} (D\phi)^2 + h_{\mu\nu} \frac{V(\phi)}{2\alpha_1} \\ + \frac{1}{(2\alpha_1)^2} \left[\frac{1}{3} \tau\tau_{\mu\nu} - (\tau\tau)_{\mu\nu} + h_{\mu\nu} \left(\frac{1}{2} (\tau\tau) - \frac{1}{12} \tau^2 - \frac{1}{2} \tau\tau_\phi + \frac{3}{4} \tau_\phi^2 \right) \right] = 0$$

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higher order & conclusions

Friedmann equations in first order dilaton gravity

- ▶ the effective equations on the brane derived ... so what?
 - ↪ let's see what can happen to the physics
- ▶ phenomenological applications to cosmology
 - *Friedmann equations modified / generalized*
- ▶ standard cosmology ↪ FRW metric tensor ansatz
 - Friedmann equations are given by gravitational equations of motion
 - ▶ trace
 - ▶ (t,t) component
 - ↪ let's just do the very same here

modified Friedmann equations: comments and complaints?

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\dot{a}}{a} \dot{\phi} - \frac{1}{3} \ddot{\phi} + \frac{1}{6} \dot{\phi}^2 + \frac{1}{3\alpha_1} V(\phi) + \frac{1}{(4\alpha_1)^2} \left[-\frac{2}{3} (\tau_1^1)^2 - 2(\tau_4^4)^2 + \frac{4}{3} \tau_1^1 \tau_\phi + 4\tau_4^4 \tau_\phi - 2\tau_\phi^2 \right] = 0$$

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{1}{3} E_1^1 - \frac{2}{3} \frac{\dot{a}}{a} \dot{\phi} + \frac{1}{12} \dot{\phi}^2 + \frac{1}{6\alpha_1} V(\phi) + \frac{1}{(4\alpha_1)^2} \left[\frac{1}{3} (\tau_1^1)^2 - \frac{2}{3} \tau_1^1 \tau_4^4 - (\tau_4^4)^2 + \frac{2}{3} \tau_1^1 \tau_\phi + 2\tau_4^4 \tau_\phi - \tau_\phi^2 \right] = 0$$

- ▶ part with the scalar factor only: exactly the same
- ▶ however, there are obviously quite relevant differences
 - ▶ **terms associated with dilaton** appear, as well as **mixing terms**
 - ↪ due to the introduction of the additional field interacting with graviton
 - ▶ **terms quadratic in the brane localized terms**
 - ↪ a feature of higher-dimensional theories?
 - ▶ $E_{\mu\nu}$: **influence of the original higher-dimensional theory**
 - ↪ on the effective 4-dimensional phenomenology

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effective gravitational equations on the brane: arbitrary N

- ▶ derivation procedure for the effective gravitational equations on the brane
 - established
 - ↪ works fine, we've just seen the example of $N = 1$ & $d = 5$
- ▶ and what about higher orders?
 - exactly the same procedure, namely
 - ▶ take $T_{\mu\nu}^{\parallel\parallel} = 0$
 - ▶ enter the solution of the brane limit of bulk equations system
 - ↪ $\{(h\mathcal{L}_n K) - (KK)\}$ & $\{\mathcal{L}_n^2 \phi - a^{\theta} \nabla_{\theta} \phi\}$ & $\{\mathcal{L}_n K_{\mu\nu} - (KK)_{\mu\nu}\}$
 - ▶ enter the result of junction conditions analysis
 - ↪ $[K_{\mu\nu}]_+$ & $[\mathcal{L}_n \phi]_+$
 - ↪ explicit results. . . even fancy notation not always sufficient

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conclusions

- ▶ starting point: **higher order dilaton gravity**
 - natural to consider in higher-dimensional space-times
 - ↪ physically viable equations of motion: *constructed*
 - ↪ appropriate Lagrangian density: *presented*
- ▶ **effective gravitational equations on the brane** (co-dimension 1)
 - derivation procedure for arbitrary order N : *established*
 - ↪ *details* and *results* presented explicitly for $N = 1$ & $d = 5$
 - ↔ together with a *cosmological example*
 - ↪ **modified Friedmann equations** on 4d brane