The Thermal Abundance of Semi-Relativistic Relics

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July 6, 2009
PASCOS 2009 @ DESY, Hamburg

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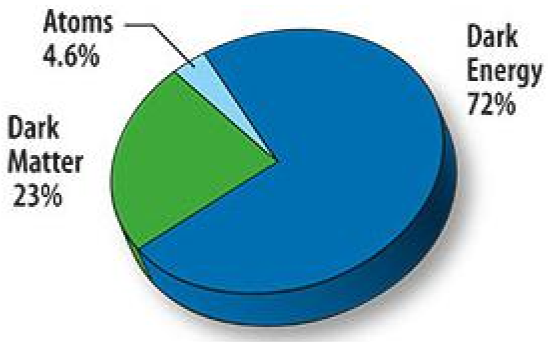
Ref:
- arXiv:0904.3046

1. Motivation



[http://wmap.gsfc.nasa.gov]

- Estimating cosmological abundances is crucial in understanding nature
e.g. particle dark matter (DM)

- Numerical calculation is needed in evaluating the relic density in many cases

⇨ **Analytic methods should be developed in various cases**

- Approximate analytical solutions are established for particles that are either relativistic or non-relativistic at decoupling

- **No analytical formula for the relic density of particles that are semi-relativistic at decoupling**

Outline

This work

- Analytic treatment that connects
  the relativistic and non-relativistic cases
- Possibility of semi-relativistic dark matter
- Late entropy production by decaying semi-relativistic relics

1. Motivation
2. Thermal abundance (review)
3. Abundance of semi-relativistic relics
4. Semi-relativistic dark matter?
5. Entropy production by decaying particles
6. Summary

2. Thermal abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann eq.:

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\rm eff} v \rangle (n_\chi^2 - n_{\chi,{\rm eq}}^2)$$

$n_{\chi,({\rm eq})}$: (Equilibrium) number density

H : Hubble parameter

$\langle \sigma v \rangle$:Thermally averaged annihilation cross section times velocity

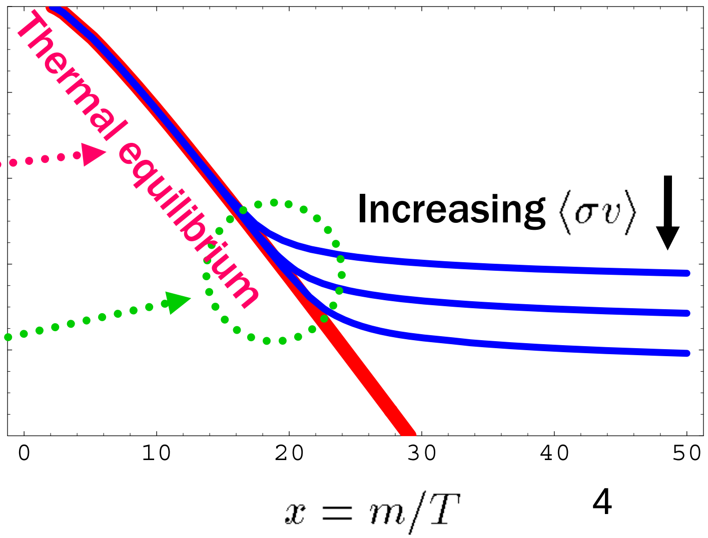
- Co-moving number density: $Y_\chi = n_\chi / s$

- At high temperatures:

Interaction rate $\Gamma = n_\chi \langle \sigma v \rangle$ $>$ Hubble expansion rate $H = \frac{\pi T^2}{M_{\rm Pl}} \sqrt{\frac{90}{g_*}}$

- When $\Gamma < H$, the number density is fixed

Decoupling T_F $(x_F = m/T_F)$



3. Thermal abundance of semi-relativistic relics

- Relativistic case: $$\Omega_\chi h^2 = 8\times 10^{-2}\frac{g_{\rm eff}}{g_*(x_F)}\left(\frac{m_\chi}{1\ {\rm eV}}\right)$$

Non-relativistic case: $$\Omega_\chi h^2 \simeq 0.1\times\left(\frac{a+3b/x_F}{10^{-9}\ {\rm GeV}^{-2}}\right)^{-1}\left(\frac{x_F}{22}\right)\left(\frac{g_*}{90}\right)^{-1/2}$$

We want a simple analytic treatment that describes the transition from non-relativistic to relativistic relics

- Assume the Maxwell-Boltzmann distribution:

$$Y_{\chi,{\rm eq}} \equiv \frac{n_{\chi,{\rm eq}}}{s} = 0.115\frac{g_\chi}{g_{*s}}x^2K_2(x)$$ ($K_n(x)$: modified Bessel function)

⇒ Thermal average of cross section σ :

$$\langle\sigma v\rangle = \frac{1}{8m_\chi^4 T K_2^2(m_\chi/T)}\int_{4m_\chi^2}^\infty {\rm d}s\ \sigma(s-4m_\chi^2)\sqrt{s}\ K_1(\sqrt{s}/T)$$

Ansatz for approximate cross sections

- Annihilation cross section:

$$\sigma v^{\mathrm{S-wave}} = \frac{G^2 s}{16\pi}$$

$$\sigma v^{\mathrm{P-wave}} = \frac{G^2 s v^2}{16\pi}$$

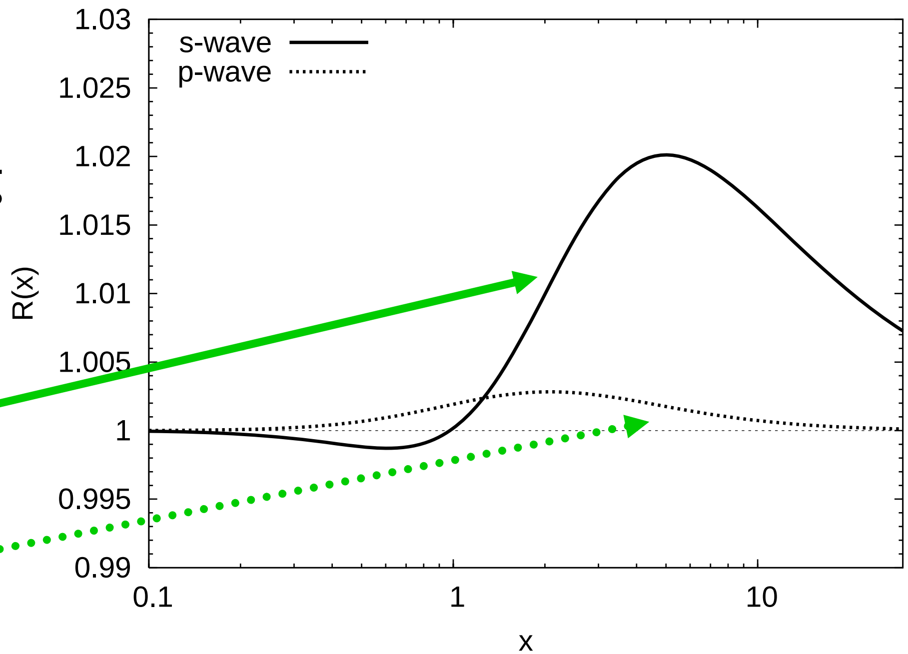
G : Effective dimension-six coupling

- Ansatz for the thermally-averaged annihilation cross section:

$$\langle \sigma v \rangle_{\mathrm{app}}^{\mathrm{S-wave}} = \frac{G^2 m_\chi^2}{16\pi} \left(\frac{12}{x^2} + \frac{5+4x}{1+x} \right)$$

$$\langle \sigma v \rangle_{\mathrm{app}}^{\mathrm{P-wave}} = \frac{G^2 m_\chi^2}{16\pi} \left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2} \right)$$

- $\langle \sigma v \rangle_{\mathrm{app}} / \langle \sigma v \rangle_{\mathrm{exact\ MB}}$:



The approx. cross sections reproduce the exact results with accuracy of a few %

Approximate abundance of semi-relativistic relics

- Define the freeze–out temperature by

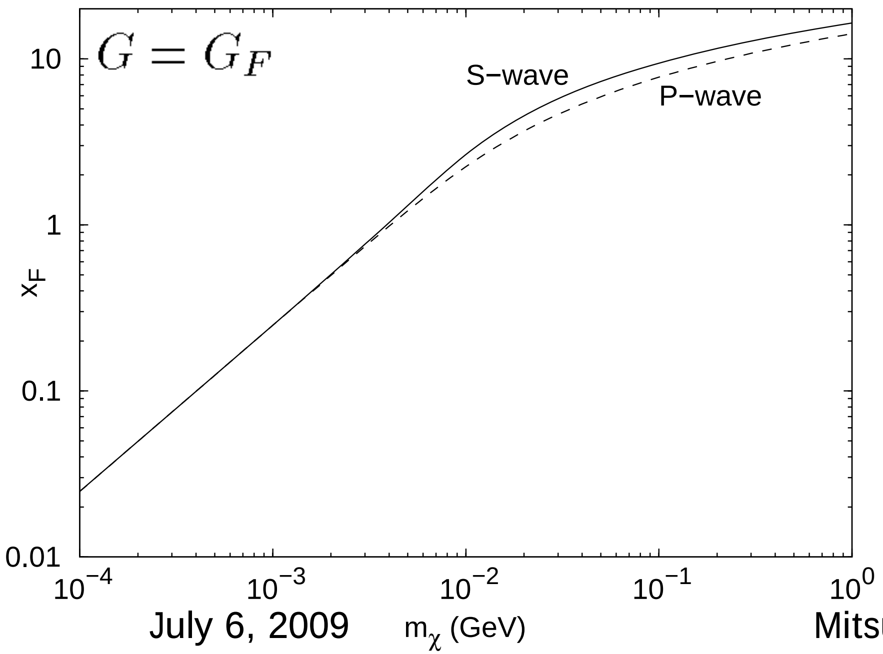
$$n_{\chi,\mathrm{eq}}(x_F)\langle\sigma v\rangle(x_F) = H(x_F)$$

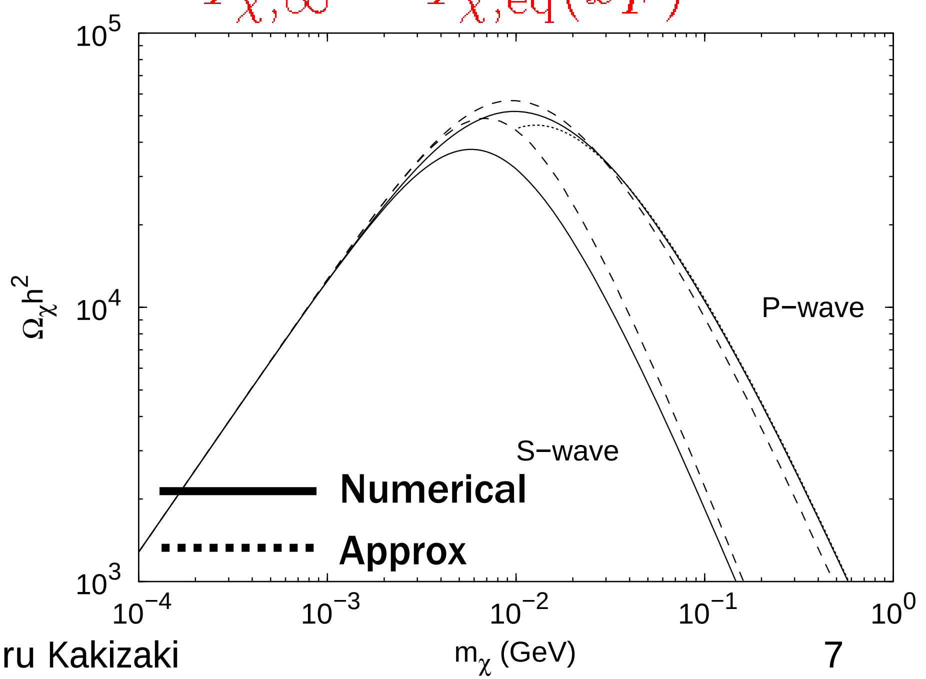
(different from the standard x_F)

- Assume the relic abundance does not change after decoupling

➡ Final abundance:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F)$$





4. Semi-relativistic dark matter?

- Observed dark matter abundance: $\Omega_{\rm DM} h^2 \simeq 0.1$

  Final density of a semi-relativistic particle: $Y_{\chi,\rm eq}(x \simeq 3) \sim 10^{-2}$

  ⇨ $m_\chi \sim 100 \ {\rm eV}, \quad T_F \sim \mathcal{O}(10) \ {\rm eV}$

- Effective dimension-six coupling is large: $G \sim 10^3 \ {\rm GeV}^{-2}$

- Possible annihilation channel: $\chi\bar{\chi} \to \nu\bar{\nu}(\nu_R\bar{\nu}_R)$

  w/ the mass of the exchange particle $< 30 \ {\rm MeV}$

  ⇨ Additional d.o.f during BBN

  Conflicts with the results of high energy experiments

⇨ **Semi-relativistically decoupling particle should decay**

4. Entropy production by decaying particles

- Suppose decaying particles dominate the energy of Universe

  By out-of-eq. decay, large amount of entropy is produced:

$$\frac{S_f}{S_i} = g_*^{1/4} \frac{m_\chi Y_{\chi,i} \tau_\chi^{1/2}}{M_{\rm Pl}^{1/2}} \propto \Omega_\chi h^2$$

[Steinhardt, Turner (1983)]

  ⇒ Overproduced particles (e.g neutralino, gravitino) can be diluted to an acceptable level

- Late decay of non-relativistic particles
  - Very large mass, very long lifetime needed for large entropy production

- Late decay of semi-relativistic particles
  - The abundance at decoupling is large

  ⇒ Significant entropy can be produced even if the mass is small

Example: sterile neutrino

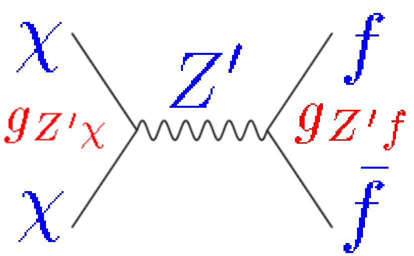
- Introduce a sterile neutrino mixed with an active neutrino (mixing angle: θ)

➡ Decay rate of the sterile neutrino:

$$\Gamma_\chi = \frac{G_F^2 m_\chi^5}{192\pi^3}\sin^2\theta, \qquad \frac{G_F m_\chi^3}{16\pi}\sin^2\theta$$

(for small m_χ) (for large m_χ)

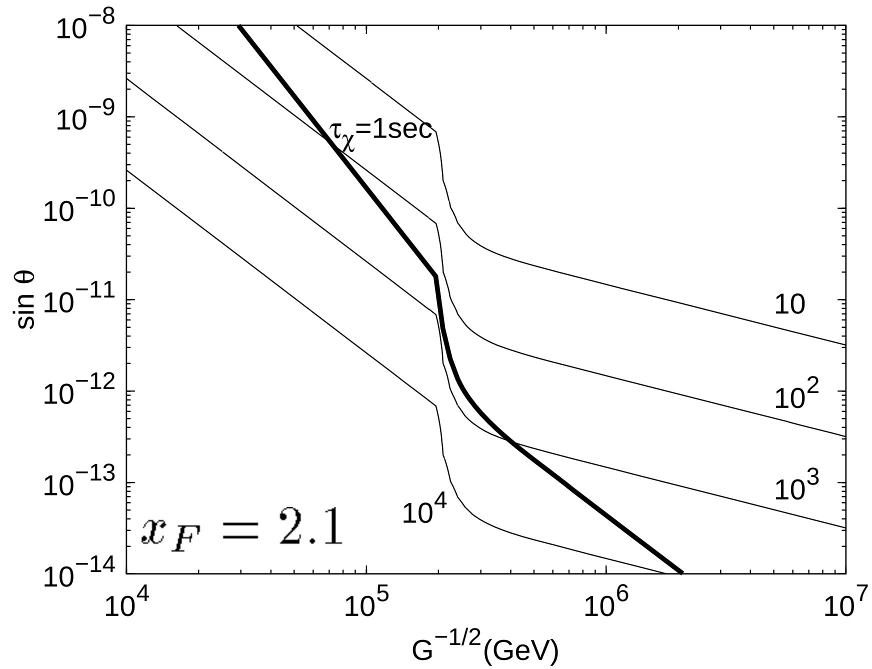
- By introducing a new heavy particle, Z' such sterile neutrinos can be thermalized



$$G = \frac{g_{Z'\chi}^2 g_{Z'f}^2}{M_Z'^4}$$

➡ Semi-relativistic decoupling is possible

- Entropy production S_f/S_i



5. Summary

- We find an approximate analytic method for the thermal abundance of relics that decoupled semi-relativistically

- It is difficult for such semi-relativistic relics to form dark matter

- Decaying semi-relativistic relics can dilute the density of other unwanted relics

Backup slides

Thermal abundance

- Relativistic relics (decouple for $x_F < 3$):

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s}$$ almost constant

⇨ Final abundance is insensitive to the freeze out temperature:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F) = \frac{45}{2\pi^4}\frac{g_\chi}{g_{*s}(x_F)}$$

- Non-relativistic relics (decouple for $x_F > 3$):

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2),$$

$$n_{\chi,\mathrm{eq}} = g_\chi \ (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\Omega_\chi h^2 = 2.7 \times 10^8 \ Y_\chi \left(\frac{m_\chi}{1\ \mathrm{GeV}}\right)$$

⇨ $$\Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9}\ \mathrm{GeV}^{-2}}\right)^{-1} \left(\frac{x_F}{22}\right) \left(\frac{g_*}{90}\right)^{-1/2} \sim \Omega_{\mathrm{DM}} h^2$$

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

→ **Non-baryonic dark matter: $\Omega_{\rm DM}h^2 = 0.1143 \pm 0.0034$**

Physics beyond the standard model (SM) of particle physics necessary

- Weakly interacting massive particles (WIMPs) χ are good candidates for dark matter (DM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,\rm standard}h^2 \sim 0.1$

- Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for WIMPs as DM particle:
  - $\chi = \bar{\chi}$, single production of χ is forbidden
- WIMP abundance n_χ is determined by the Boltzmann eq.:

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

$H = \dot{R}/R$: Hubble expansion parameter

$\langle \sigma v \rangle$: thermal average of the annihilation cross section $\sigma(\chi\chi \to \mathrm{SM\ particles})$ times relative velocity v

$n_{\chi,\mathrm{eq}}$: equilibrium number density

- Introduce $Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, x = \frac{m_\chi}{T}$

$$\Rightarrow \frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx} (Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)$$