How fast do (electroweak) bubbles move?

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collaboration with Miguel Sopena

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Relics of the phase transition

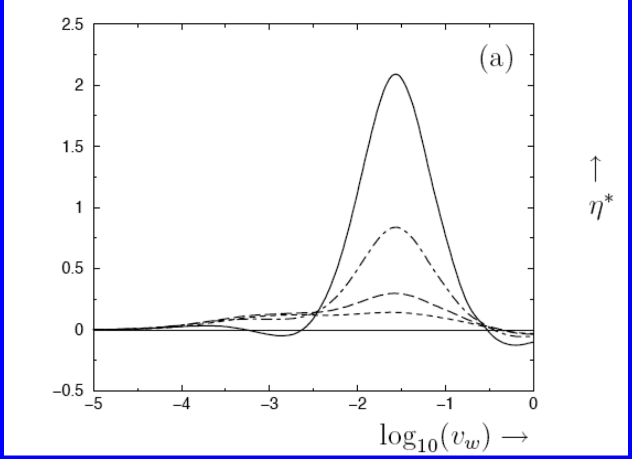
- gravitational waves

dependence on the wall velocity!

$$h_0^2\Omega_{\rm det} \simeq 1.2 \times 10^{-7}\kappa^2 \langle R\rangle^2 H_*^2 \left[\frac{\alpha}{\alpha+1}\right]^2 \left[\frac{v_b^2}{0.24+v_b^3}\right]^2 \left[\frac{100}{g_*}\right]^{1/3}.$$

Kamionkowski, Kosowsky, Turner '93

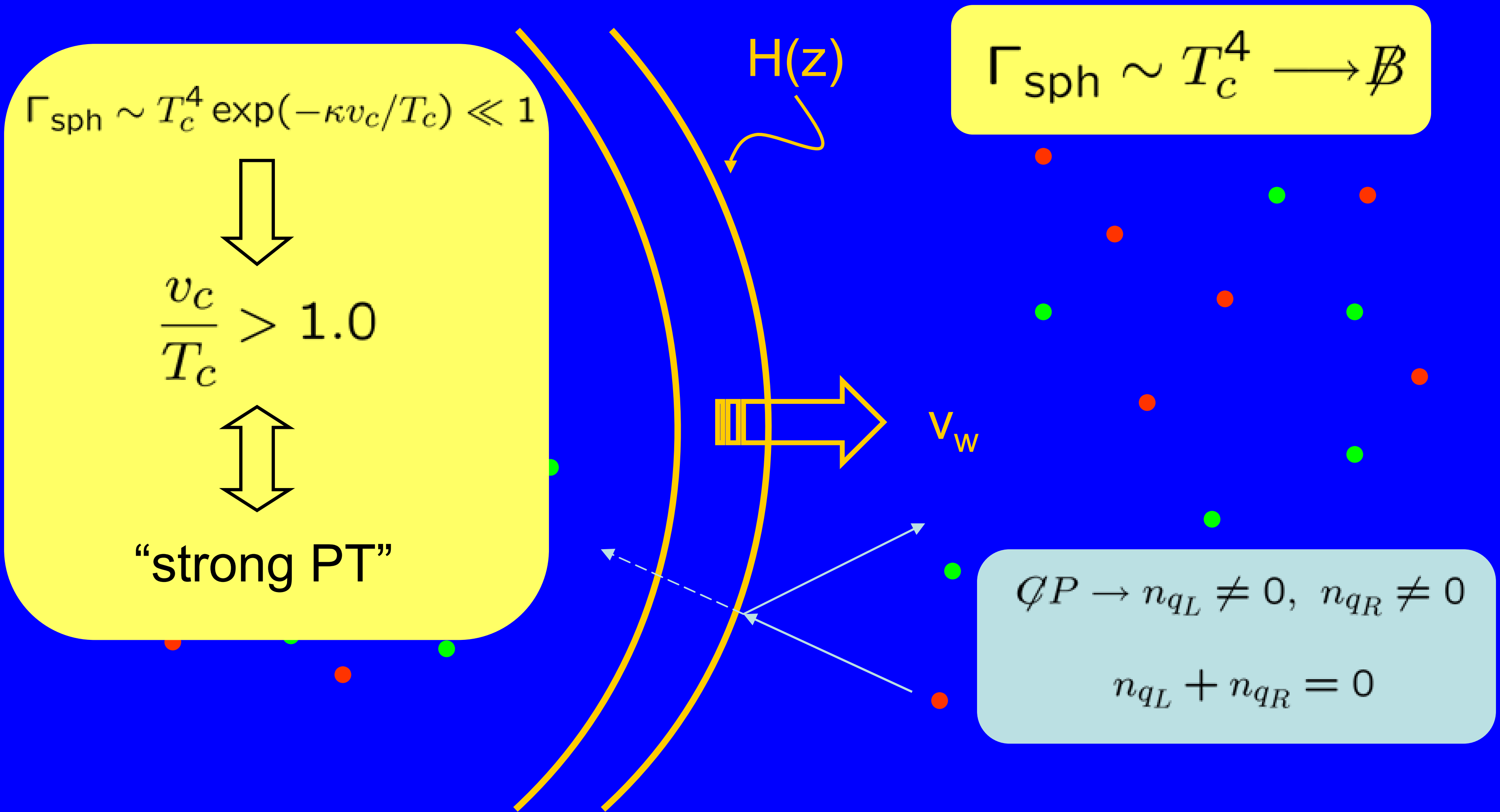
- baryon asymmetry



SH, Schmidt '00

- magnetic fields

Electroweak baryogenesis



1) The strength of the PT

Thermal potential (or lattice):

$$V(H,T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4$$

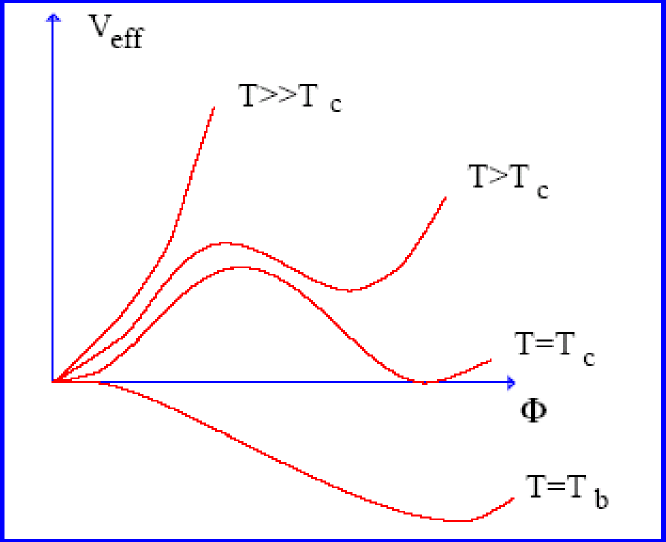
- Boson loops:

  SM: gauge bosons

  SUSY: light stops

  2HDM: heavy Higgses

- tree-level: extra singlets: $\lambda S H^2$, NMSSM, etc.
- replace H^4 by H^6, etc.



SM + higher-dim. operators

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{M^2}|H|^6$$

Zhang '93

Grojean et al. '04

maybe related to strong dynamics at the TeV scale, such as technicolor or gravity?

(or simply comes from integrating out extra scalars)

two parameters, (λ, M) $\leftrightarrow$ (m_h, M)

λ can be negative $\rightarrow$ bump because of $|H|^4$ and $|H|^6$

$$\begin{aligned} V_{\rm eff}(\phi,T) = \quad & \frac{1}{2}\left(-\mu^2 + \left(\frac{1}{2}\lambda + \frac{3}{16}g_2^2 + \frac{1}{16}{g_1}^2 + \frac{1}{4}y_t^2\right)T^2\right)\phi^2 \\ & - \frac{g_2^3}{16\pi}T\phi^3 + \frac{\lambda}{4}\phi^4 + \frac{3}{64\pi^2}y_t^4\phi^4 \ln\left(\frac{Q^2}{c_F T^2}\right) \\ & + \frac{1}{8M^2}(\phi^6 + 2\phi^4 T^2 + \phi^2 T^4). \end{aligned}$$

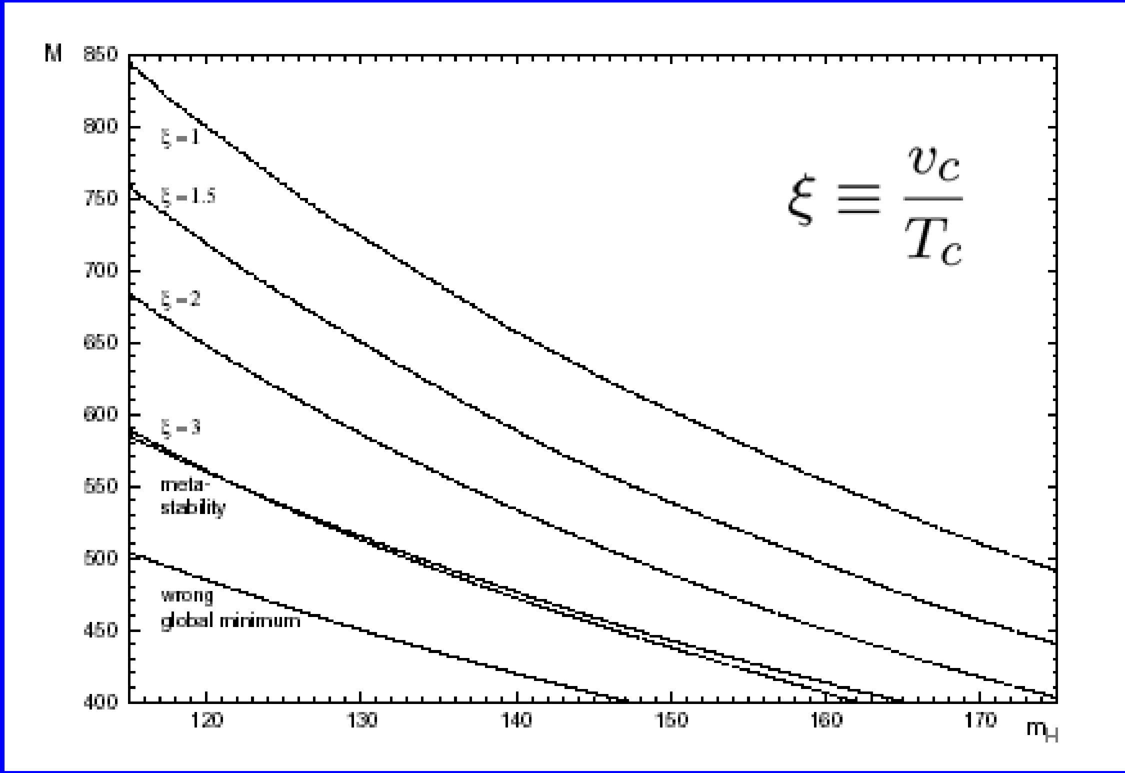
Results for the PT

Evaluating the 1-loop thermal potential:

strong phase transition for M<850 GeV up to m_h ~170 GeV

(LEP bound applies, m_h >114 GeV)

wall thickness $2 < L_w T_c < 16$



Bödeker, Fromme, S.H., Seniuch '04

Similar results, including Higgs cubic terms

Delaunay, Grojean, Wells '07

2) Dynamics of the transition

At the critical temperature T_c the two minima are degenerate

Bubble nucleation starts at $T< T_c$ with a rate

$$\Gamma = A\,T^4\,e^{-S_3/T},$$

Where the bubble energy is

$$S_3 = 4\pi \int dr\, r^2 \left[\frac{1}{2}\left(\frac{d\varphi}{dr}\right) + V(\varphi, T)\right]$$

The bubble configuration follows from

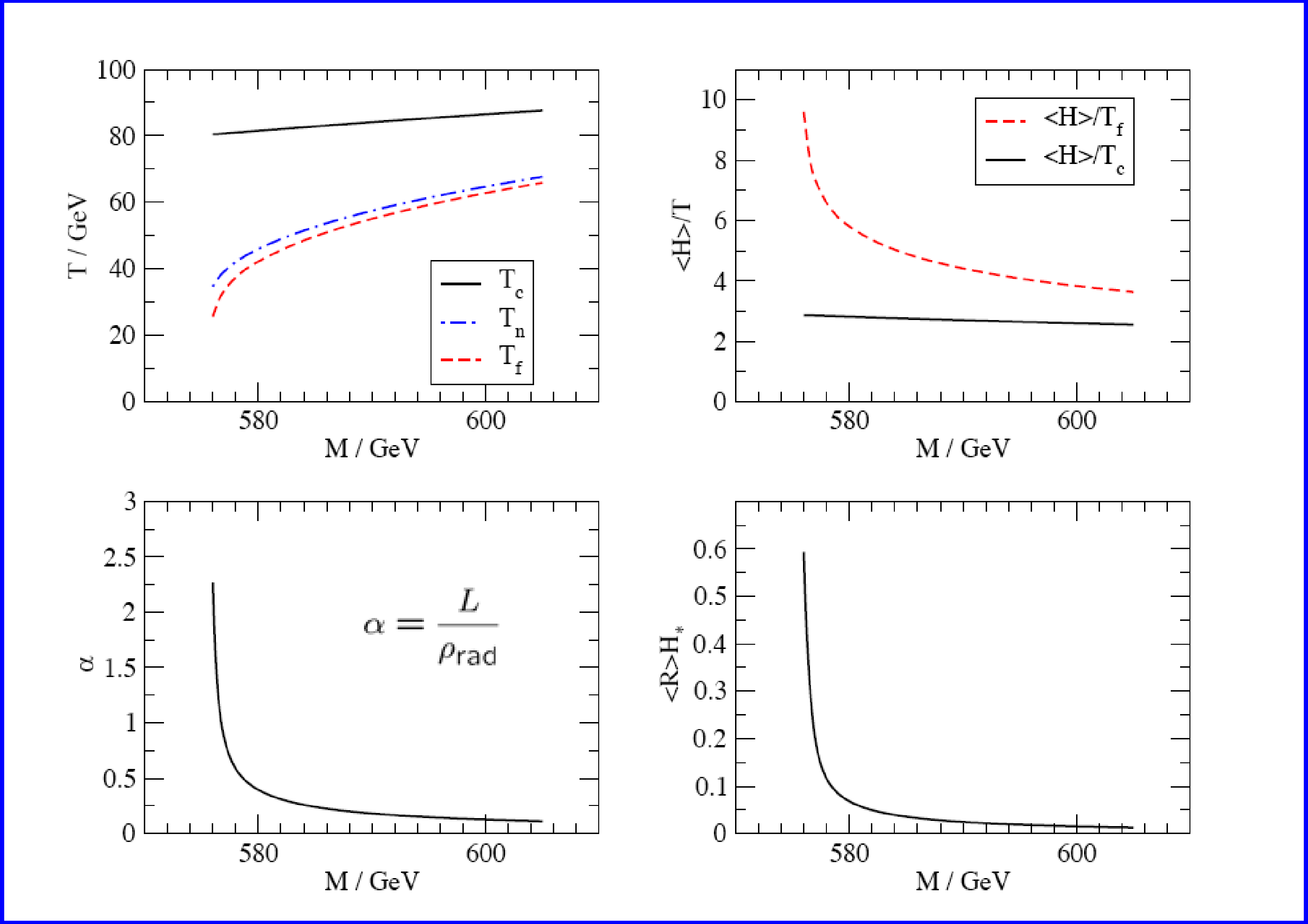
(with appropriate BC's)

$$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} = V'(\phi)$$

This bounce solution is a saddle point, not a minimum
$\rightarrow$ difficult to compute for multi field models (one field: shooting)

For a algorithm see Konstandin, S.H. '06

Key parameteres of the phase transition: Φ^6 model, m_h=120 GeV



S. H. & Konstandin '07

Compute as function of temperature: bubble configurations $\rightarrow$ E

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{M^2}|H|^6$$

nucleation rate Γ~exp(-E)

bubble distribution $\rightarrow$ R

The wall velocity:

Friction with the plasma balances the pressure

Distinguish: supersonic vs. subsonic (v_s^2=1/3)

Standard model: v_b~ -0.35 - 0.45 for low Higgs masses [Moore, Prokopec '95]

MSSM: v_b~0.05 [John, Schmidt '00]

All other models: no detailed computations

For very strong phase transitions: bubbles become supersonic, velocity dominated by hydrodynamics (neglect friction) [Steinhardt '82]

$$v_b(\alpha) = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1+\alpha}.$$

(for sufficiently large α > few %?)

When does this fail ??

**Recently: can the walls run away? [Bodeker, Moore '09]

How to compute the wall velocity?

Main ingredients: pressure difference vs. plasma friction

Also important: reheating due to release of latent heat

Microscopic description: Moore, Prokopec '95

$$(\partial_t + \dot{z}\partial_z + \dot{p}_z \partial_{p_z}) f = \mathcal{C}[f]$$

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3\, 2E} \delta f(p,x) = 0$$

$$f = \frac{1}{1+\exp\frac{E - E\delta T/T - p_z v - \mu}{T}}$$

(fluid ansatz)

$$\begin{aligned}
a v_w \frac{\mu'}{T} + v_w \frac{\delta T'}{T} + \frac{1}{3} v' + F_1 &= -\Gamma_{\mu 1} \frac{\mu}{T} - \Gamma_{T1} \frac{\delta T}{T} \\
b v_w \frac{\mu'}{T} + v_w \frac{\delta T'}{T} + \frac{1}{3} v' + F_2 &= -\Gamma_{\mu 2} \frac{\mu}{T} - \Gamma_{T2} \frac{\delta T}{T} \\
b \frac{\mu'}{T} + \frac{\delta T'}{T} + v_w v' + 0 &= -\Gamma_v v
\end{aligned}$$

$$F_1 = -\frac{v_w \ln 2}{9\zeta_3} \frac{(m^2)'}{T^2}, \qquad F_2 = -\frac{v_w \zeta_2}{42\zeta_4} \frac{(m^2)'}{T^2}$$

(force terms)

$$(F = \dot{p}_z = -\partial_z E(z, p_z))$$

→ Complicated set of coupled field equations

and Boltzmann equations

need many scattering rates

SM: v ~ 0.35 - 0.45

Simplified approach: (Ignatius, Kajantie, Kurki-Suonio, Laine ‘94)

1) describe friction by a friction coefficient $1/\Gamma$

2) Model the fluid by a fluid velocity and temperature

$$\phi''(x) = \frac{\partial V}{\partial \phi} + \frac{v\gamma}{\Gamma}\phi'(x)$$

$$(4aT^4 - T\frac{\partial V}{\partial T})\gamma^2 v = \text{const.}$$

$$(4aT^4 - T\frac{\partial V}{\partial T})\gamma^2 v^2 + aT^4 + \frac{1}{2}\phi'(x)^2 - V = \text{const.}$$

$\leftarrow$ 1 + 1 dimensions

3) Determine Γ from fitting the to the full result by Moore and Prokopec

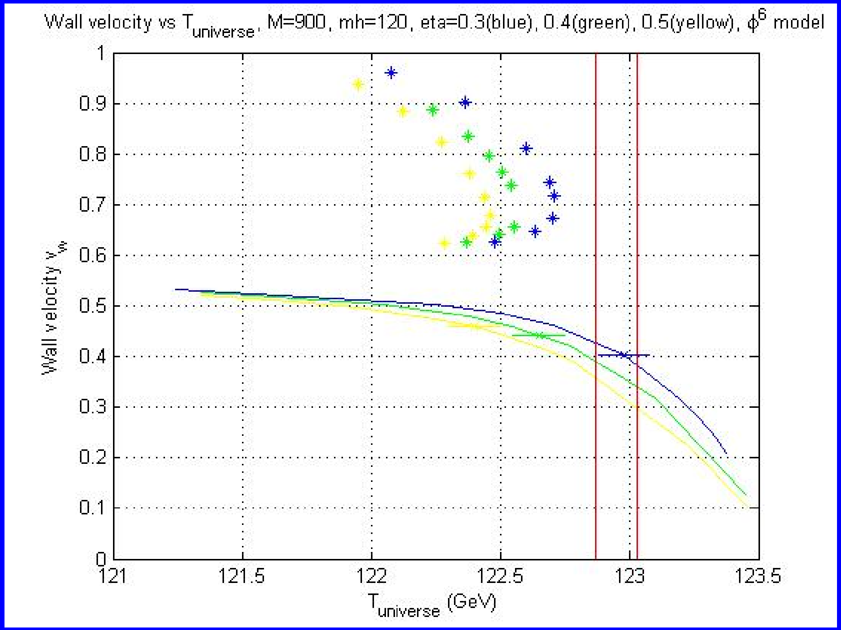
(with Miguel Sopena, see also Megevand, Sanchez ‘09)

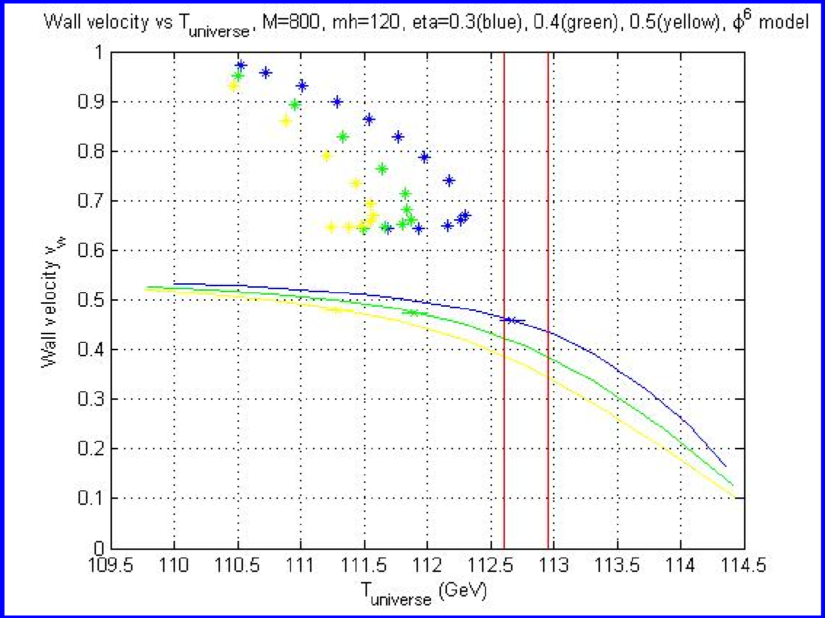
We find: a good fit with a universal Γ is possible

$\rightarrow$ the formalism should describe situations with SM friction well

$\rightarrow$ study models with SM friction, but different potential, e.g. phi^6 model

Results for Φ^6 model





- for a moderately strong phase transition, the wall moves subsonic in the dim-6 model (good for baryon asymmetry, bad for GW's)
- instable bubbles for some temperatures?

Results for the PT

Evaluating the 1-loop

thermal potential:

strong phase transition

for M<850 GeV

up to m_h ~170 GeV

(LEP bound applies,

m_h >114 GeV)

wall thickness
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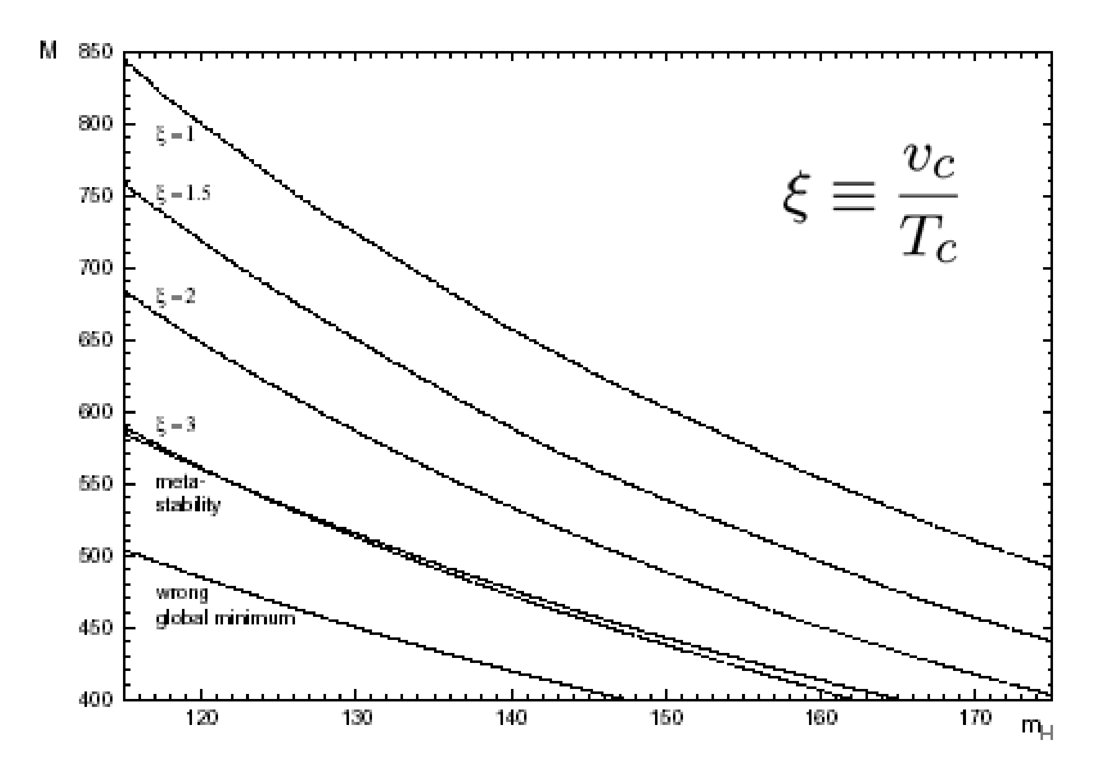

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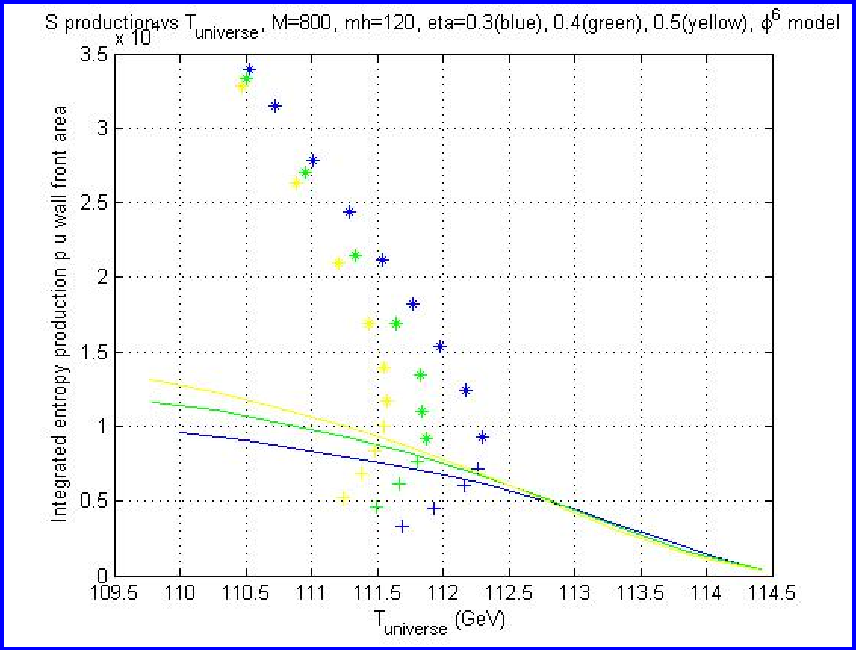

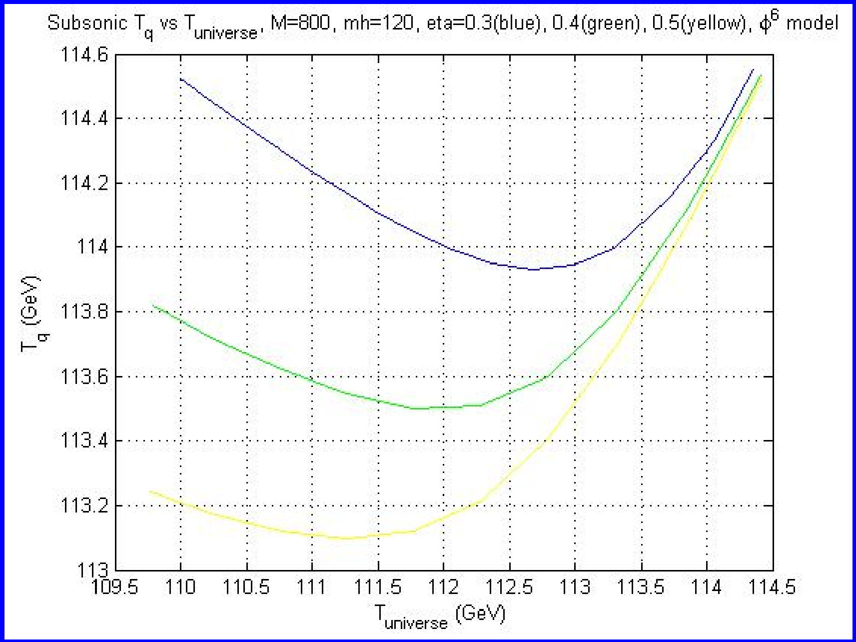

Detonations, if possible, give the biggest entropy

Instability due to large reheating, effect of 1+1?

summary

- Simplified way to compute wall velocities with SM-like friction
- for a moderately strong phase transition, the wall moves subsonic in the dim-6 model

- major uncertainties:
  - what friction constant to take for very strong phase transitions?
  - effect of infrared gauge field condenstate?
  - instabilities, turbulence?

aim: use this model to do 3d numerical simulations of the electroweak phase transition (e.g. for GW production)