Gauge Threshold Corrections for Local String Models

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Local vs Global

Model-building in string theory can either be local or global.

Global models:
- Canonical example is weakly coupled heterotic string.
- Model specification requires global consistency conditions.
- Relies on geometry of entire compact space
- Limit $V \rightarrow \infty$ also gives $\alpha_{SM} \rightarrow 0$: cannot separate string and Planck scales.
- Other examples: IIA/IIB intersecting brane worlds, M-theory on G2 manifolds
Local vs Global

Local models:

▸ Canonical example: branes at singularity

▸ Model specification only requires knowledge of local geometry and local tadpole cancellation.

▸ Full consistency depends on existence of a compact embedding of the local geometry.

▸ Standard Model gauge and Yukawa couplings remain finite in the limit $\mathcal{V} \to \infty$.

It is possible to have $M_P \gg M_s$ by taking $\mathcal{V} \to \infty$.

▸ Examples: LARGE volume models, branes at singularities, IIB/F-theory GUTs.
Local and Global Models
Threshold Corrections: Supergravity
Threshold Corrections: String Theory
Threshold Corrections: Local GUTs
Conclusion

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Threshold corrections $\Delta_a(M, \tilde{M})$ are difference between naive and actual gauge coupling running:

$$\left. \frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2} \right|_0 + \beta_a \ln \left( \frac{M_s^2}{\mu^2} \right) + \Delta_a(M, \tilde{M}) .$$

Arise from heavy KK/string/winding states.
Threshold Corrections in Supergravity

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

\[ g_{phys}^{-2}(\Phi, \Phi, \mu) = \text{Re}(f_a(\Phi)) + \frac{b_a}{16\pi^2} \ln \left( \frac{M_P^2}{\mu^2} \right) + \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \Phi, \mu) + \frac{\sum_r n_r T_a(r) - T(G)}{16\pi^2} \hat{K}(\Phi, \Phi) - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \Phi, \mu). \]

(Holomorphic coupling)

(β-function running)

(NSVZ term)

(Kähler-Weyl anomaly)

(Konishi anomaly)

Relates measurable couplings and holomorphic couplings.
For local models in IIB

- Kähler potential $\hat{K}$ is given by

$$\hat{K} = -2 \ln \mathcal{V} + \ldots$$

- Matter kinetic terms $\hat{Z}$ are given by

$$\hat{Z} = \frac{f(\tau_{SM})}{\mathcal{V}^{2/3}}$$

Why? When we decouple gravity the physical couplings

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{\hat{Z}_\alpha \hat{Z}_\beta \hat{Z}_\gamma}}$$

should remain finite and be $\mathcal{V}$-independent.
\[ \hat{K} = -2 \ln \mathcal{V}, \quad \hat{Z} = \frac{f(\tau_{SM})}{\mathcal{V}^{2/3}} \]

- Local models require a LARGE bulk volume (\( \mathcal{V} \sim 10^4 \) for \( M_s \sim M_{GUT} \), \( \mathcal{V} \sim 10^{15} \) for \( M_s \sim 10^{11} \) GeV).
- Kähler and Konishi anomalies are formally one-loop suppressed. However if volume is LARGE, both anomalies are enhanced by \( \ln \mathcal{V} \) factors.
- This implies the existence of large anomalous contributions to physical gauge couplings!
Plug in $\hat{K} = -2 \ln \mathcal{V}$ and $\hat{Z} = \frac{1}{\mathcal{V}^{2/3}}$ into Kaplunovsky-Louis formula.

We restrict to terms enhanced by $\ln \mathcal{V}$ and obtain:

\[
g^{-2}_{phys}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \frac{\left(\sum_r n_r T_a(r) - 3 T_a(G)\right)}{8 \pi^2} \ln \left(\frac{M_P}{\mathcal{V}^{1/3} \mu}\right)
\]

\[
g^{-2}_{phys}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln \left(\frac{(RM_s)^2}{\mu^2}\right)
\]

- Gauge couplings start running from an effective scale $RM_s$ rather than $M_s$.
- Universal $\text{Re}(f_a(\Phi))$ implies unification occurs at a super-stringy scale $RM_s$ rather than $M_s$.
- Want to check and understand this in string theory!
Threshold Corrections in String Theory

- Calculate using the **background field method**.
- Running gauge couplings are the 1-loop coefficient of
  \[
  \frac{1}{4g^2} \int d^4x \sqrt{g} F^a_{\mu\nu} F^{a,\mu\nu}
  \]
- Turn on background magnetic field \( F_{23} = B \) and compute the quantised string spectrum.
- Use the string partition function to compute the 1-loop vacuum energy
  \[
  \Lambda = \Lambda_0 + \frac{1}{2} \left( \frac{B}{2\pi^2} \right)^2 \Lambda_2 + \frac{1}{4!} \left( \frac{B}{2\pi^2} \right)^4 \Lambda_4 + \ldots
  \]
- From \( \Lambda_2 \) term we can extract beta function running and threshold corrections.
String theory 1-loop vacuum function given by partition function

\[ \Lambda_{1\text{-}loop} = \frac{1}{2}(T + KB + A(B) + MS(B)). \]

- Require \( \mathcal{O}(B^2) \) term of this expansion.
- Background magnetic field only shifts moding of open string states.
- Torus and Klein Bottle amplitudes do not couple to open strings.
- Only annulus and Möbius strip amplitudes contribute at \( \mathcal{O}(B^2) \).
We want examples of calculable local models with non-zero beta functions.

- The simplest such examples are (fractional) D3 branes at orbifold/orientifold singularities.
- String can be exactly quantised and all calculations can be performed explicitly.

We have studied

- D3 branes on $\mathbb{C}^3/\mathbb{Z}_4$, $\mathbb{C}^3/\mathbb{Z}_6$, $\mathbb{C}^3/\mathbb{Z}_6'$, $\mathbb{C}^3/\Delta_{27}$.
- D3/D7 systems on $\mathbb{C}^3/\mathbb{Z}_3$.
- D3/O3 on $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_4\}$, $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_6\}$, $\mathbb{C}^3/\{I\Omega, \mathbb{Z}_6'\}$.
We separately evaluate each amplitude in the $\theta^N$ sector (e.g. $\mathbb{Z}_4$)

$$A(B) = \int_0^\infty \frac{dt}{2t} \text{Str} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} \frac{q(p^\mu p_\mu + m^2)/2}{2} \right)$$

$$M(B) = \int_0^\infty \frac{dt}{2t} \text{Str} \left( \frac{(1 + \theta + \theta^2 + \theta^3)}{4} \frac{1 + (-1)^F}{2} \frac{\Omega p^\mu p_\mu + m^2)/2}{2} \right)$$

\[ \theta^0 = (1, 1, 1) \text{ is an } \mathcal{N} = 4 \text{ sector.} \]

\[ \theta^1 = (1/4, 1/4, -1/2) \text{ and } \theta^3 = (-1/4, -1/4, 1/2) \text{ are } \mathcal{N} = 1 \text{ sectors.} \]

\[ \theta^2 = (1/2, 1/2, 0) \text{ is an } \mathcal{N} = 2 \text{ sector.} \]

The amplitudes reduce to products of Jacobi $\vartheta$-functions with different prefactors.
$N = 1$ amplitudes

$$A_{N=1}^{(k)} = - \int \frac{dt}{2t} \frac{1}{(2\pi^2 t)} \sum_{\alpha, \beta=0,1/2} \frac{\eta_{\alpha\beta}}{2} \text{Tr} \left( \gamma_{\theta^k} \otimes \gamma_{\theta^k}^{-1} \frac{i(\beta_1 + \beta_2)}{2\pi^2} \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta \\ \end{array} \right]}{\vartheta \left[ \begin{array}{c} 1/2 \\ 1/2 \\ \end{array} \right]} \left( \frac{iet}{2} \right) \right)$$

$$\times \prod_{i=1}^{3} \left( -2 \sin \pi \theta_i^k \right) \frac{\vartheta \left[ \begin{array}{c} \alpha \\ \beta + \theta_i^k \\ \end{array} \right]}{\vartheta \left[ \begin{array}{c} 1/2 \\ 1/2 + \theta_i^k \\ \end{array} \right]}.$$
\[ N = 2 \text{ amplitudes} \]

\[
A_{N=2}^{(k)} = - \int \frac{dt}{2t} \frac{1}{2\pi^2 t} \sum_{\alpha, \beta = 0, 1/2} \frac{\eta_{\alpha\beta}}{2} (-1)^{2\alpha} \text{Tr} \left( \gamma_{\theta^k} \otimes \gamma_{\theta^k}^{-1} \frac{i(\beta_1 + \beta_2)}{2\pi^2} \vartheta \left[ \frac{\alpha}{\beta} \right] \left( \frac{i\epsilon t}{2} \right) \right) \\
\times \vartheta \left[ \frac{\alpha}{\beta} \right] \prod_{i=1}^{2} \frac{2 (-2 \sin \pi \theta^k_i)}{\eta^3} \vartheta \left[ \frac{\alpha}{\beta + \theta^k_i} \right] \\
\times \frac{\eta^3}{\prod_{i=1}^{2} \vartheta \left[ \frac{1/2}{1/2 + \theta^k_i} \right]}.
\]

\[
M_{N=2}^{(k)} = 2 \int \frac{dt}{2t} \frac{1}{2\pi^2 t} \sum_{\alpha, \beta = 0, 1/2} \frac{\eta_{\alpha\beta}}{2} (-1)^{2\alpha} \text{Tr} \left( \frac{i}{2\pi^2} \beta \gamma_{\Omega^k} \gamma_{\Omega^k}^{-T} \vartheta \left[ \frac{\alpha}{\beta} \right] \left( \frac{i\epsilon t}{2} \right) \right) \\
\times \vartheta \left[ \frac{\alpha}{\beta} \right] \prod_{i=1}^{2} \frac{2 (-2 \sin \pi R^k_i)}{\eta^3} \vartheta \left[ \frac{\alpha}{\beta + R^k_i} \right] \\
\times \frac{\eta^3}{\prod_{i=1}^{2} \vartheta \left[ \frac{1/2}{1/2 + R^k_i} \right]}.
\]
$\mathcal{N} = 1$ sector

$$
\mathcal{A}_{\mathcal{N}=1} + \mathcal{M}_{\mathcal{N}=1} \quad \xrightarrow{t \to \infty} \quad \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \beta^{\mathcal{N}=1}_a
$$

$$
\mathcal{A}_{\mathcal{N}=1} + \mathcal{M}_{\mathcal{N}=1} \quad \xrightarrow{t \to 0} \quad \int \frac{dt}{t^2} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \left( 0 + \mathcal{O}(e^{-1/t}) \right).
$$

- In IR $t \to \infty$ limit, obtain contribution of $\mathcal{N} = 1$ sectors to $\beta$-functions.
- In UV $t \to 0$ limit, amplitudes vanish as local tadpole cancellation is enforced.
- Field theory running cut off at $M_s$ as string states contribute to amplitude.
\( \mathcal{N} = 2 \) sector

\[
\mathcal{A}_{\mathcal{N}=2} + \mathcal{M}_{\mathcal{N}=2} \quad \overset{t\to\infty}{\longrightarrow} \quad \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \beta_{\mathcal{N}=2}^a \\
\mathcal{A}_{\mathcal{N}=2} + \mathcal{M}_{\mathcal{N}=2} \quad \overset{t\to0}{\longrightarrow} \quad \int \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 \times \beta_{\mathcal{N}=2}^a
\]

- In IR \( t \to \infty \) limit, obtain contribution of \( \mathcal{N} = 2 \) sectors to \( \beta \)-functions.
- In UV \( t \to 0 \) limit, amplitudes unaltered.
- Consequence of \( \mathcal{N} = 2 \) susy: all open string oscillators non-BPS and decouple.
- How should we interpret this?
\[ \mathcal{N} = 2 \text{ sector} \]

\[ \int_{1/\mu^2}^{1/\infty^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a^{\mathcal{N}=2} \]

- Divergence in \( t \to \infty \) limit is physical: this is the IR limit and we recover ordinary \( \beta \)-function running.

- Divergence in \( t \to 0 \) limit is unphysical: open string UV limit and this amplitude must be finite in a consistent string theory.

- Physical understanding of the divergence is best understood from closed string channel.
$\mathcal{N} = 2$ sector

Annulus amplitude:

```
B X B
```

Annulus amplitude in $t \to 0$ limit:

```
B X B
```
$\mathcal{N} = 2$ sector

- $t \to 0$ divergence corresponds to a source for a partially twisted RR 2-form.
- In the local model this propagates into the bulk of the Calabi-Yau.
- In a compact model this becomes a physical divergence and tadpole must be cancelled by bulk sink branes/O-planes.
- Tadpole is sourced locally but cancelled globally.
$\mathcal{N} = 2$ sector

The purely local computation omits the following worldsheets:
The purely local string computation includes all open string states for $t > 1/(RM_s)^2$, i.e. $M < RM_s$.

However for $t < 1/(RM_s)^2$ we must include new winding states in the partition function.

These are essential for global consistency but are omitted by a purely local computation.

Threshold corrections become finite

$$\int_{1/\infty^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a \rightarrow \int_{1/(RM_s)^2}^{1/\mu^2} \frac{dt}{2t} \frac{1}{4} \left( \frac{B}{2\pi^2} \right)^2 b_a$$

Effective UV cutoff is actually $RM_s$ and not $M_s$!
Summary and Matching Field Theory

Running takes the form

\[
\frac{1}{g^2(\mu)} = \left. \frac{1}{g^2} \right|_0 + \beta_a \ln \left( \frac{M_s^2}{\mu^2} \right) + \beta_a^{\mathcal{N}=2} \ln \left( \frac{(RM_s)^2}{M_s^2} \right). 
\]

- $\mathcal{N} = 1$ sectors run from $M_s$ into the IR.
- $\mathcal{N} = 2$ sectors run from $RM_s$ into the IR.

How to reconcile with Kaplunovsky-Louis?

\[
g_{\text{phys}}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln \left( \frac{(RM_s)^2}{\mu^2} \right). 
\]
Summary and Matching Field Theory

- At the singularity holomorphic gauge couplings are
  \[ f_a = S + s_{ak} M_k \]
- \( M_k \) is the twisted blow-up field.
- At 1-loop,
  \[ M_k \rightarrow M_k + \frac{\alpha}{16\pi^2} \ln \mathcal{V} \]
  and \( \langle M_k \rangle \neq 0 \).
- Coefficients \( s_{ak} \) are \( \mathcal{N} = 1 \) contributions to running \( \beta^{\mathcal{N}=1}_{ak} \).
Summary and Matching Field Theory

Running takes the form

\[
\frac{1}{g^2(\mu)} = \frac{1}{g^2} \big|_0 + (\beta^{N=1}_a + \beta^{N=2}_a) \ln \left( \frac{M_s^2}{\mu^2} \right) + \beta^{N=2}_a \ln \left( \frac{(R M_s)^2}{M_s^2} \right).
\]

Kaplunovsky-Louis:

\[
g^{-2}_{phys}(\Phi, \bar{\Phi}, \mu) = S + \beta^{N=1}_a T + (\beta^{N=1}_a + \beta^{N=2}_a) \ln \left( \frac{(R M_s)^2}{\mu^2} \right).
\]

\(T\) obtains a one-loop vev at the singularity:

\[
\langle T \rangle = - \ln \left( \frac{(R M_s)^2}{M_s^2} \right).
\]
Summary and Matching Field Theory

For D3 @ orbifold singularities, there are no $\mathcal{N} = 1$ contributions to $\beta$ functions (vanishing of twisted tadpoles).

- Gauge coupling unification occurs at $RM_s$.

For D3/D7 @ orbifold singularities, 33 and 37 worldsheets combine to give $\mathcal{N} = 1$ contribution.

- In general no gauge coupling unification, for $\mathbb{Z}_3$ singularity unification at $M_s$.

For D3 @ orientifold singularities, annulus and Mobius worldsheets give $\mathcal{N} = 1$ contribution.

- Running starts at $RM_s$, with non-universal shift at $M_s$: no gauge coupling unification.
Local GUTs

- Proposal to realise $SU(5)$ GUTs through branes on del Pezzo with hypercharged flux.
- Flux is quantised on cycle that is non-trivial in $H^2(dP, \mathbb{Z})$ but trivial in $H^2(CY, \mathbb{Z})$.
- Holomorphic gauge couplings are
  
  $f_{SU(3)} = T_{dP} + h_3(F)S + \epsilon_3(U),$
  $f_{SU(2)} = T_{dP} + h_2(F)S + \epsilon_2(U),$
  $f_{U(1)_Y} = T_{dP} + h_1(F)S + \epsilon_Y(U).$

- For gauge unification assume $h_3 = h_2 = h_1$ and $\epsilon_i$ is negligible.
Local GUTs

\[ CY_3 \]

\[ dP_n \]

\[ R |_S \]

\[ F_Y \]
Local GUTs: Field Theory

\[ f_{SU(3)} = f_{SU(2)} = f_Y = T + h(F)S. \]

\[ g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \text{Re}(f_a(\Phi)) + \beta_a \ln \left( \frac{(RM_s)^2}{\mu^2} \right). \]

- Kaplunovsky-Louis implies gauge unification occurs at \( RM_s \) rather than \( M_s \).
- Field redefinitions are universal and cannot alter universality of gauge kinetic functions or affect result.
- Bulk does not decouple and enters the unification scale.
Local GUTs: String Theory

Can understand this from the string perspective:

- \( T_{dP} \) is an ‘\( \mathcal{N} = 1 \)’ sector: associated to \( SU(5) \) universal physics.

- GUT breaking comes from hypercharge flux associated to an ‘\( \mathcal{N} = 2 \)’ sector.

- Locally \( \mathcal{F}_Y \) sources tadpole divergence via

  \[
  \int_{dP} C_{2,Y} \wedge \mathcal{F}_Y
  \]

- Globally \( C_{2,Y} \) is absent: divergence regulated at scale \( R M_s \) and gauge couplings run to this scale.
Local GUTs: Warping and Exotica

Can apply ideas to warped throats:

- Suppose GUT realised on del Pezzo down a warped throat.
- Hypercharge cycle trivialises outside throat.
- Finiteness of threshold corrections requires knowledge of tadpole cancellation.
- In closed string channel string must leave throat, in open string channel need to include string states reaching out of throat and into bulk - with mass $M_P$.
- Running continues until Planck scale - possibility of TeV warped throat with gauge unification close to $M_P$?
Conclusions

- Lots of interesting physics associated to running gauge couplings for local models.
- For $\mathcal{N} = 1$ sectors, gauge couplings run from $M_s$.
- For $\mathcal{N} = 2$ sectors, gauge couplings run from $RM_s$.
- For local GUTs with hypercharge flux breaking, unification scale is $RM_s$ and not $M_s$.

Bulk cannot be consistently decoupled from the gauge theory.